Popular Astronomy.

Vol. XIII No. 7. AUGUST AND SEPTEMBER, 1905. Whole No. 127

THE DETERMINATION OF ORBITS.

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Let us look at the chief things to be considered in finding the orbit of a body moving around the Sun according to the law of gravitation. We omit the small corrections for parallax and the aberration of light. As the body moves in a conic section and we assume the mass of the body to be zero, there will be generally six parts, or elements, to determine. Hence we must have six given data, which will be furnished by three observations of the right ascension and declination of the body. It is convenient to convert these into geocentric longitudes and latitudes. The body will be seen at three given times on three straight lines passing through the center of the Earth and the body. Let x, y, z, be the rectangular heliocentric co-ordinates of the body; X, Y, O, those of the Earth at the first observation. Let the distance of the body from the Earth be Δ, and the geocentric longitude and latitude α and β. The curtate distance $\rho = \Delta \cos \beta$. We have the equations,

$$\begin{aligned} x - X &= \Delta \cos \beta \cos \alpha &&= \rho \cos \alpha \\ y - Y &= \Delta \cos \beta \sin \alpha &&= \rho \sin \alpha \\ z - O &= \Delta \sin \beta &&= \rho \tan \beta \end{aligned}$$

The co-ordinates of the Earth are known, and also α and β. Our three equations contain four unknown quantities, x, y, z, and Δ or ρ.

If the distance were known we should have the radius vector by the equation,

$$r^2 = x^2 + y^2 + z^2$$

Inversely if r were known we could find Δ. This distance, or ρ, appears therefore as an important factor of the problem. If we distinguish the quantities for the other observations by one and two accents, the same conditions will be repeated. We must look to other parts of the question to find means to overcome the difficulty. By Kepler's law the body describes in its orbit areas proportional to the times. We know the times of the observations and to these we turn for aid in the solution of the

problem. If v, v', v'', are the true anomalies, the double areas of the three triangles will be

$$rr' \sin (v' - v) : \qquad r'r'' \sin (v'' - v') : \qquad rr'' \sin (v'' - v) :$$

Denote these double areas by $[rr']$; $[r'r'']$; $[rr'']$. These areas are not exactly proportional to the times, since they differ from the conic sectors, but they are nearly so if the intervals of time are small. See Gauss, Th. M. arts. 88-109. As the positions of the body are in a plane passing through the origin of co-ordinates, or the center of the Sun, we have

$$\begin{aligned} Ax + By + Cz &= 0. \\ Ax' + By' + Cz' &= 0. \\ Ax'' + By'' + Cz'' &= 0. \end{aligned}$$

Eliminating the constants A, B, C, we find

$$xy''z' - xy'z'' + x'yz'' - x'y''z + x''y'z - x''yz' :$$

Generally this is six times the volume of a tetrahedron whose vertex is at the center of the Sun, and which stands on these points. As the points are in a plane the volume is zero. The expression may be written,

$$\begin{aligned} x(y''z' - y'z'') + x'(yz'' - y''z) + x''(y'z - yz') &= 0 \\ y(x'z'' - x''z') + y'(x''z - xz'') + y''(xz' - x'z) &= 0 \\ z(x''y' - x'y'') + z'(xy'' - x''y) + z''(x'y - xy') &= 0 \end{aligned}$$

The quantities enclosed in parentheses in the first of these equations are the projections of the triangles in the orbit plane on the plane of yz. In the second equation the triangles are projected on the plane of xz; and in the third equation they are projected on the plane of xy. The cosines of projection will divide out, and if we put

$$[rr'] = rr' \sin (v' - v) : \quad [rr''] = rr'' \sin (v'' - v) : \quad [r'r''] = r'r'' \sin (v'' - v') :$$

we shall have

$$\begin{aligned} [r'r'']x - [rr'']x' + [rr']x'' &= 0. \\ [r'r'']y - [rr'']y' + [rr']y'' &= 0. \\ [r'r'']z - [rr'']z' + [rr']z'' &= 0. \end{aligned}$$

(Th. M. 112.) Let the polar co-ordinates of the Earth be R and L, etc. By substituting in the last equations and putting

$$n = \frac{[r'r'']}{[rr'']} : \qquad n'' = \frac{[rr']}{[rr'']} :$$

since

$$x = \rho \cos \alpha + R \cos L : \qquad y = \rho \sin \alpha + R \sin L : \qquad z = \rho \tan \beta :$$

etc., we shall have the fundamental equations:

(a.)

$$n.(\rho \cos \alpha + R \cos L) - (\rho' \cos \alpha' + R' \cos L') + n''(\rho'' \cos \alpha'' + R'' \cos L'') = 0.$$
$$n.(\rho \sin \alpha + R \sin L) - (\rho' \sin \alpha' + R' \sin L') + n''(\rho'' \sin \alpha'' + R'' \sin L'') = 0.$$
$$n.\ \rho \tan \beta \qquad - \rho' \tan \beta' \qquad + n''\ \rho'' \tan \beta'' \qquad = 0.$$

We can eliminate ρ and ρ'' from these equations, and find ρ' in terms of n, n'', and known quantities. Multiply the first equation by

$$\tan \beta \sin \alpha'' - \tan \beta'' \sin \alpha,$$

the second by $\tan \beta'' \cos \alpha - \tan \beta \cos \alpha''$,

the third by $\sin(\alpha - \alpha'')$,

and the sum of the products gives

$$\begin{aligned}[\tan \beta \sin (\alpha'' - \alpha') - \tan \beta' \sin (\alpha'' - \alpha) + \tan \beta'' \sin (\alpha' - \alpha)] \,.\, \rho' \\ = n\,R.\,[\tan \beta \sin (\alpha'' - L) - \tan \beta'' \sin (\alpha - L)] \\ + n''\,R.''\,[\tan \beta \sin (\alpha'' - L'') - \tan \beta'' \sin (\alpha - L'')] \\ - R.'\,[\tan \beta \sin (\alpha'' - L') - \tan \beta'' \sin (\alpha - L')]\end{aligned}$$

When the intervals of time are small the coefficient of ρ' is small, and care must be given to computing this coefficient. To simplify the equation for ρ' Encke draws a great circle from the first to the third position of the body. (Th. M. 138.) Call the longitude of the ascending node of this circle K, and its inclination J. From the right triangle we have

$$\tan \beta = \sin (\alpha - K) \tan J$$
$$\tan \beta'' = \sin (\alpha'' - K) \tan J$$

Let w be any angle, then

$$\begin{aligned}&\sin (\alpha'' - w) \sin (\alpha - K) - \sin (\alpha - w) \sin (\alpha'' - K) \\ &= \tfrac{1}{2} \cos [(\alpha'' - \alpha) - w + K] - \tfrac{1}{2} \cos [(\alpha'' - \alpha) + w - K] \\ &= \sin (\alpha'' - \alpha) \sin (w - K),\end{aligned}$$

and by substituting the values of $\tan \beta$ and $\tan \beta''$, the equation becomes

$$\begin{aligned}&[\sin (\alpha'' - \alpha) \sin (\alpha' - K) \tan J - \sin (\alpha'' - \alpha) \tan \beta'] \,.\, \rho' \\ &= n\,R \sin (\alpha'' - \alpha) \sin (L - K) \tan J - R' \sin (\alpha'' - \alpha) \sin (L' - K) \tan J \\ &+ n''\,R'' \sin (\alpha'' - \alpha) \sin (L'' - K) \tan J.\end{aligned}$$

Let $\tan \beta^\circ = \sin (\alpha' - K) \tan J$, and dividing out the common factor $\sin (\alpha'' - \alpha)$, we find

$$\begin{aligned}(\tan \beta^\circ - \tan \beta) \,.\, \rho' &= n\,R \sin (L - K) \tan J - R' \sin (L' - K) \tan J \\ &+ n''\,R'' \sin (L'' - K) \tan J,\end{aligned}$$

or

$$\frac{\sin (\beta' - \beta^\circ)}{\cos \beta^\circ \cos \beta' \tan J} \,.\, \rho' = R' \sin (L' - K) - n\,R \sin (L - K) - n''\,R'' \sin (L'' - K)$$

The quantity $\sin (\beta' - \beta^\circ)$ is small generally, and must be computed with care. For computing $\tan J$ and K see (Th. M. 78), or add and subtract the equations, then

$$\tan J \sin [\tfrac{1}{2}(\alpha'' + \alpha) - K] = \frac{\tan \beta'' + \tan \beta}{2 \cos \tfrac{1}{2}(\alpha'' - \alpha)},$$

$$\tan J \cos [\tfrac{1}{2}(\alpha'' + \alpha) - K] = \frac{\tan \beta'' - \tan \beta}{2 \sin \tfrac{1}{2}(\alpha'' - \alpha)},$$

J is less than 90°.

In the value of ρ' we have the quantaties n and n''. We might take for these the ratios of the intervals of time which have been observed, but since ρ' has the small factor, $\sin(\beta' - \beta^\circ)$, these ratios might not give a sure approximation to the true values. We must examine the ratios of the areas of the triangles and the times. In order to do this we reduce the co-ordinates of the first and third observations to the middle time. From the laws of motion we have, k being the Gaussian constant,

$$\int_t^{t'} r^2 dv = k(t' - t) \cdot \sqrt{p} = \tau.'' \sqrt{p}$$

$$\int_t^{t''} r^2 dv = k(t'' - t) \cdot \sqrt{p} = \tau.' \sqrt{p}$$

$$\int_{t'}^{t''} r^2 dv = k(t'' - t') \cdot \sqrt{p} = \tau. \sqrt{p}$$

p being the semi-parameter, and $\tau' = \tau + \tau''$; $d\tau = k \cdot dt$. By Taylor's theorem

$$x = x' - \frac{dx'}{d\tau}\tau'' + \frac{1}{1.2} \cdot \frac{d^2x'}{d\tau^2}\tau''^2 - \frac{1}{1.2.3} \cdot \frac{d^3x'}{d\tau^3} \cdot \tau''^3 + \text{etc.}$$

$$y = y' - \frac{dy'}{d\tau}\tau'' + \frac{1}{1.2} \cdot \frac{d^2y'}{d\tau^2}\tau''^2 - \frac{1}{1.2.3} \cdot \frac{d^3y^1}{d\tau^3} \cdot \tau''^3 + \text{etc.}$$

$$x'' = x' + \frac{dx'}{d\tau} \cdot \tau + \frac{1}{1.2} \cdot \frac{d^2x'}{d\tau^2}\tau^2 + \frac{1}{1.2.3} \cdot \frac{d^3x'}{d\tau^3} \cdot \tau^3 + \text{etc.}$$

$$y'' = y' + \frac{dy'}{d\tau} \cdot \tau + \frac{1}{1.2} \cdot \frac{d^2y'}{d\tau^2}\tau^2 + \frac{1}{1.2.3} \cdot \frac{d^3y'}{d\tau^3} \cdot \tau^3 + \text{etc.}$$

The equations of motion in the plane of the orbit are

$$\frac{d^2x}{d\tau^2} + \frac{x}{r^3} = 0. \qquad \frac{d^2y}{d\tau^2} + \frac{y}{r^3} = 0.$$

Hence we find

$$\frac{d^3x}{d\tau^3} = \frac{3x}{r^4} \cdot \frac{dr}{d\tau} - \frac{1}{r^3} \cdot \frac{dx}{d\tau}.$$

$$\frac{d^4x}{d\tau^4} = \left(\frac{1}{r^6} - \frac{12}{r^5}\left(\frac{dr}{d\tau}\right)^2 + \frac{3}{r^4} \cdot \frac{d^2r}{d\tau^2}\right)x + \frac{6}{r^4} \cdot \frac{dr}{d\tau} \cdot \frac{dx}{d\tau}.$$

The values of the y derivatives are similar. If we put

$$a = 1 - \tfrac{1}{2}\frac{\tau''^2}{r'^3} - \tfrac{1}{2}\frac{\tau''^3}{r'^4}\cdot\frac{dr'}{d\tau} + \left(\frac{1}{r'^6} - \frac{12}{r'^5}\left(\frac{dr'}{d\tau}\right)^2 + \frac{3}{r'^4}\cdot\frac{d^2r'}{d\tau^2}\right)\cdot\frac{\tau''^4}{24} + \text{etc.}$$

$$b = \tau'' - \frac{1}{6}\frac{\tau''^3}{r'^3} - \tfrac{1}{4}\cdot\frac{\tau''^4}{r'^4}\cdot\frac{dr'}{d\tau} + \ldots\ldots$$

$$a'' = 1 - \tfrac{1}{2}\frac{\tau^2}{r'^3} + \tfrac{1}{2}\frac{\tau^3}{r'^4}\cdot\frac{dr'}{d\tau} + \left(\frac{1}{r'^6} - \frac{12}{r'^5}\cdot\left(\frac{dr'}{d\tau}\right)^2 + \frac{3}{r'^4}\cdot\frac{d^2r'}{d\tau^2}\right)\cdot\frac{\tau^4}{24} + \text{etc.}$$

$$b'' = \tau - \frac{1}{6}\frac{\tau^3}{r'^3} + \tfrac{1}{4}\frac{\tau^4}{r'^4}\cdot\frac{dr'}{d\tau} + \ldots\ldots$$

we may write

$$x = ax' - b\cdot\frac{dx'}{d\tau}; \qquad x'' = a''x' + b''\frac{dx'}{d\tau};$$

$$y = ay' - b\cdot\frac{dy'}{d\tau}; \qquad y'' = b''y' + b''\cdot\frac{dy'}{d\tau};$$

Hence the double areas become

$$[rr'] = xy' - x'y = b\left(x'\frac{dy'}{d\tau} - y'\frac{dx'}{d\tau}\right).$$

$$[r'r''] = x'y'' - x''y' = b''\left(x'\frac{dy'}{d\tau} - y'\frac{dx'}{d\tau}\right).$$

$$[rr''] = xy'' - x''y = (ab'' + a''b)\cdot\left(x'\frac{dy'}{d\tau} - y'\frac{dx'}{d\tau}\right).$$

But $xdy - ydx = r^2dv = \sqrt{p}\cdot d\tau$.

and therefore

$$[rr'] = b\cdot\sqrt{p}; \qquad [r'r''] = b''\sqrt{p},$$

$$[rr''] = (ab'' + a''b)\cdot\sqrt{p},$$

Since we can reduce by the relation $\tau' = \tau + \tau''$, we have in terms of the times,

$$[rr'] = \tau''\sqrt{p}\cdot\left\{1 - \frac{1}{6}\frac{\tau''^2}{r'^3} - \tfrac{1}{4}\cdot\frac{\tau''^3}{r'^4}\cdot\frac{dr'}{d\tau} + \ldots\ldots\right\}$$

$$[r'r''] = \tau\sqrt{p}\cdot\left\{1 - \frac{1}{6}\frac{\tau^2}{r'^3}\cdot + \tfrac{1}{4}\frac{\tau^3}{r'^4}\cdot\frac{dr'}{d\tau} + \ldots\ldots\right\}$$

$$[rr''] = \tau'\sqrt{p}\cdot\left\{1 - \frac{1}{6}\frac{\tau'^2}{r'^3} + \tfrac{1}{4}\frac{\tau'^2(\tau - \tau'')}{r'^4}\cdot\frac{dr'}{d\tau} + \ldots\ldots\right\}$$

Let us see how the areas of the triangles are connected with the elements of the orbit. (Th. M. 82.) If e be the excentricity of the ellipse, and v the true anomaly, the polar equation gives,

$$\frac{p}{r} = 1 + e\cos v \qquad (v = u - \pi),$$

$$\frac{p}{r'} = 1 + e\cos v'$$

$$\frac{p}{r''} = 1 + e\cos v''$$

Multiply the first of these equations by $\sin(v''-v')$, the second by $\sin(v-v'')$, and the third by $\sin(v'-v)$, and add the products, (Th. M. 78),

$$\frac{p}{r}\sin(v''-v') - \frac{p}{r'}\sin(v''-v) + \frac{p}{r''}\sin(v'-v)$$
$$= \sin(v''-v) - \sin(v''-v) + \sin(v'-v),$$

since the quantity multiplied by e reduces to zero. Now

$$\sin(v''-v') + \sin(v'-v) = 2\sin\tfrac{1}{2}(v''-v)\cos\tfrac{1}{2}(v''-2v'+v)$$

and $$-\sin(v''-v) = -2\sin\tfrac{1}{2}(v''-v)\cos\tfrac{1}{2}(v''-v).$$

The right side becomes

$$2\sin\tfrac{1}{2}(v''-v)\,.\,[\cos\tfrac{1}{2}(v''-2v'+v) - \cos\tfrac{1}{2}(v''-v)],$$

and finally $$4\sin\tfrac{1}{2}(v''-v')\sin\tfrac{1}{2}(v''-v)\sin\tfrac{1}{2}(v'-v).$$

Multiply and divide the left hand of the equation by $rr'r''$, and it becomes

$$\frac{p}{rr'r''}\cdot[\,[r'r''] - [rr''] + [rr']\,]$$

Multiply and divide the right side by

$$r^2r'^2r''^2\cos\tfrac{1}{2}(v''-v')\cos\tfrac{1}{2}(v''-v)\cos\tfrac{1}{2}(v'-v)$$

and we find,

$$p = \frac{[rr']\,.\,[rr'']\,.\,[r'r'']}{[r'r''] - [rr''] + [rr']} \cdot \frac{1}{2rr'r''\cos\tfrac{1}{2}(v''-v')\cos\tfrac{1}{2}(v''-v)\cos\tfrac{1}{2}(v'-v)}$$

If p is known we can find e and the other elements. But this value of p has the small divisor $[r'r''] - [rr''] + [rr']$, which is the double area of the small triangle between the chords. If the observations are near each other the small errors of observation may give a very erroneous value of p. (Th. M. 86.) Let us examine the values of this divisor in terms of the times. From the results given above

$$[r'r''] - [rr''] + [rr'] = \sqrt{p}\,\Big\{ + \tau'' - \frac{1}{6}\frac{\tau''^3}{r'^3} - \tfrac{1}{4}\cdot\frac{\tau''^4}{r'^4}\cdot\frac{dr'}{d\tau} + \ldots\ldots$$
$$+ \tau \;\; - \frac{1}{6}\frac{\tau^3}{r'^3} + \tfrac{1}{4}\frac{\tau^4}{r'^4}\cdot\frac{dr'}{d\tau} + \ldots\ldots$$
$$- \tau' + \frac{1}{6}\frac{\tau'}{r'^3} - \tfrac{1}{4}\frac{\tau'(\tau-\tau'')}{r'^4}\cdot\frac{dr'}{d\tau} + \ldots\ldots \Big\}$$

Since $\tau' = \tau + \tau''$, the first column is zero. The next is

$$\frac{1}{6r'^3}\cdot 3(\tau^2\tau'' + \tau''^2) = \frac{1}{2r'^3}\cdot\tau\tau''.(\tau+\tau'') = \frac{1}{2r'^3}\cdot\tau\tau'\tau''.$$

The third column is

$$\tfrac{1}{4}\cdot\frac{dr'}{d\tau}\cdot\frac{1}{r'^4}\cdot[-\tau''^4 + \tau^4 - \tau'^3(\tau-\tau'')]$$

The term in brackets is

$$\begin{aligned}&(\tau - \tau'') \,.\, [(\tau + \tau'')\,(\tau^2 + \tau''^2) - \tau'^3]\\ &= (\tau - \tau'') \,.\, [\tau^2 + \tau''^2) - \tau'^2] \,.\, \tau'\\ &= (\tau - \tau'') \,.\, 2\tau\tau'\tau'' ;\end{aligned}$$

so that

$$[r'r''] - [r'r''] + [rr'] = \frac{\tau\tau'\tau''}{2r'^3} \cdot \sqrt{p} \cdot \left\{ 1 - \frac{(\tau - \tau'')}{r'} \cdot \frac{dr'}{d\tau} + \ldots \right\}$$

The small triangle is therefore a quantity of the third order, with respect to the intervals of time. If the intervals are equal the second term in the brackets is zero. For the ratios n and n'', we have by division,

$$n'' = \frac{[rr']}{[rr'']} = \frac{\tau''}{\tau'} \cdot \left\{ 1 + \frac{1}{6} \cdot \frac{\tau\,(\tau' + \tau'')}{r'^3} - \tfrac{1}{4} \cdot \frac{\tau\,(\tau^2 + \tau\tau'' - \tau''^2)}{r'^4} \cdot \frac{dr'}{d\tau} + \ldots \right\}$$

$$n = \frac{[r'r'']}{[rr'']} = \frac{\tau}{\tau'} \cdot \left\{ 1 + \frac{1}{6} \cdot \frac{\tau''\,(\tau' + \tau)}{r'^3} + \tfrac{1}{4} \cdot \frac{\tau''\,(\tau''^2 + \tau\tau'' - \tau^2)}{r'^4} \cdot \frac{dr'}{d\tau} + \ldots \right\}$$

The sum of these is

$$n + n'' = 1 + \tfrac{1}{2} \frac{\tau\tau''}{r'^3} - \tfrac{1}{2} \frac{\tau\tau''\,(\tau - \tau'')}{r'^4} \cdot \frac{dr'}{d\tau} + \ldots\ldots$$

In the equation for ρ' we have the terms,

$$n\,R \sin (L - K) + n''\,R'' \sin (L'' - K)$$

and these give, including terms of the second order,

$$\frac{\tau}{\tau'}\,R \sin (L - K) + \frac{\tau''}{\tau'}\,R'' \sin (L'' - K)$$

$$+ \frac{\tau\tau''}{6r'^3\tau'} \cdot [(\tau' + \tau)\,R \sin (L - K) + (\tau' + \tau'')\,R'' \sin (L'' - K)]$$

We can take out the first term as a factor in the second, and write,

$$\frac{\tau\,R \sin (L - K) + \tau''\,R'' \sin (L'' - K)}{\tau'} \cdot \left(1 + \frac{\tau\tau''}{2r'^3} \right)$$

$$+ \frac{\tau\tau''}{6r'^3\tau'} \cdot (\tau - \tau'') \,.\, [R'' \sin (L'' - K) - R \sin (L - K)]$$

This reduction is made by adding and subtracting

$$\frac{\tau\,R \sin (L - K) + \tau''\,R'' \sin (L'' - K)}{\tau'} \cdot \frac{\tau\tau''}{3r'^3} :$$

When the obervations are near together the above term in brackets is of the first order, so that the whole term is of the third order, and may be neglected. To show this directly we assume two arcs, the sum and difference of which are $L'' - K$, and $L - K$; thus

$$x = \tfrac{1}{2}\,(L'' + L) - K, \qquad x + y = L'' - K,$$
$$y = \tfrac{1}{2}\,(L'' - L), \qquad x - y = L - K.$$

Then

$$(R'' + R) \cos x \sin y = (R'' + R) \sin \tfrac{1}{2}(L'' - L) \cos [\tfrac{1}{2}(L'' + L) - K]$$
$$(R'' - R) \sin x \cos y = (R'' - R) \cos \tfrac{1}{2}(L'' - L) \sin [\tfrac{1}{2}[L'' + L) - K]$$

By adding and subtracting we find the form $R'' \sin (L'' - K) - R \sin (L - K)$. If we write the expression

$$n R \sin (L - K) + n'' R'' \sin (L'' - K)$$

in the form

$$\frac{R \sin (L - K) + \dfrac{n''}{n} R'' \sin (L'' - K)}{1 + \dfrac{n''}{n}} \cdot (n + n'')$$

we see that terms of the second order will be included if we take

$$\frac{n''}{n} = \frac{\tau''}{\tau} \quad \text{and} \quad n + n'' = 1 + \frac{\tau\tau''}{2r'^3};$$

since we have

$$\frac{n''}{n} = \frac{\tau''}{\tau} \cdot \left\{ 1 + \frac{1}{6} \cdot \frac{(\tau - \tau')\tau'}{r'^3} + \ldots\ldots \right\};$$

and

$$n + n'' = 1 + \frac{1}{2r'^3}\tau\tau'' \cdot \left\{ 1 - \frac{\tau - \tau''}{\tau'} \cdot \frac{dr'}{d\tau} + \ldots\ldots \right\}$$

This shows the reason of Gauss for taking

$$P = \frac{\tau''}{\tau}; \quad \text{and} \quad Q = (n + n'' - 1)\, 2r'^3 = \tau\tau'';$$

If y denote the ratio of the conic sector to the triangle we have,

$$y\,[r'r''] = \tau \sqrt{p}; \qquad y'\,[rr''] = \tau' \sqrt{p}; \qquad y''\,[rr'] = \tau'' \sqrt{p};$$

Hence,

$$\frac{\tau''}{\tau} = \frac{y''}{y} \cdot \frac{n''}{n}; \qquad \therefore \frac{n''}{n} = \frac{\tau'' y}{\tau y''} = \text{correct } P.$$

Now

$$p = \frac{[rr'] \,.\, [r'r'']}{n + n'' - 1} \cdot \frac{1}{2rr'r'' \cos \tfrac{1}{2}(v'' - v') \cos \tfrac{1}{2}(v' - v) \cos \tfrac{1}{2}(v'' - v)}$$

But

$$[rr'] \,.\, [r'r''] = \frac{\tau\tau''}{yy''} \cdot p;$$

$$(n + n'' - 1)\, 2r'^3 =$$

$$\frac{\tau\tau''}{yy''} \cdot \frac{r'^2}{rr'' \cos \tfrac{1}{2}(v'' - v') \cos \tfrac{1}{2}(v' - v) \cos \tfrac{1}{2}(v' - v)} = \text{correct } Q.$$

(Th. M. 134)

Having found ρ' and r' we may get r, r'', ρ, ρ'', by the method of Gauss or find ρ and ρ'' from the fundamental equations, (a).

In the second of these equations (a) diminish the longitudes by L', since the line of reference is arbitrary, and we have

$$n\,.\,[\rho \sin(\alpha - L') + R \sin(L - L')] - \rho' \sin(\alpha' - L') + n''\,.\,[\rho'' \sin(\alpha'' - L') + R'' \sin(L'' - L) = 0.$$

Also the third is, $n\rho \tan\beta - \rho' \tan\beta' + n''\rho'' \tan\beta'' = 0.$

Multiply first by $\tan\beta'$, and second by $\sin(\alpha' - L')$:

$$n\,.\,[\rho \tan\beta' \sin(\alpha - L') + R \tan\beta' \sin(L - L')] - \rho' \tan\beta' \sin(\alpha' - L') + n''\,.\,[\rho'' \tan\beta' \sin(\alpha'' - L') + R'' \tan\beta' \sin(L'' - L')] = 0.$$

and

$$n\rho \tan\beta \sin(\alpha' - L') - \rho' \tan\beta' \sin(\alpha' - L') + n''\rho'' \tan\beta'' \sin(\alpha' - L') = 0.$$

By subtraction

$$\left.\begin{aligned} &+n\,.\,[\rho\ \tan\beta' \sin(\alpha - L') - \rho\ \tan\beta\ \sin(\alpha' - L') + R\ \tan\beta' \sin(L - L')] \\ &+n''.\,[\rho''\tan\beta' \sin(\alpha'' - L') - \rho''\tan\beta'' \sin(\alpha' - L') + R''\tan\beta' \sin(L'' - L')] \end{aligned}\right\} = 0.$$

or

$$\rho'' = \frac{\rho n}{n''} \cdot \frac{\tan\beta \sin(\alpha' - L') - \tan\beta' \sin(\alpha - L')}{\tan\beta' \sin(\alpha'' - L') - \tan\beta'' \sin(\alpha' - L')} + \frac{R\,n \sin(L' - L) - R''\,n'' \sin(L'' - L')}{\tan\beta' \sin(\alpha'' - L') - \tan\beta'' \sin(\alpha' - L')} \cdot \frac{\tan\beta'}{n''}$$

which is a rigorous expression. Put $[RR'] = RR' \sin(L' - L)$, etc., and multiply and divide the last term by R', and it becomes,

$$\left(\frac{[r'r'']}{[rr']} - \frac{[R'R'']}{[RR']}\right) \times \frac{R \tan\beta' \sin(L' - L)}{\tan\beta' \sin(\alpha'' - L') - \tan\beta'' \sin(\alpha' - L')}$$

Olber's assumption is that

$$\frac{n}{n''} = \frac{t'' - t'}{t' - t},$$

and the last term is zero. It is generally very small, so that his method gives a good result.

In ancient times the appearance of great comets in the heavens terrified people, who thought the end of the world near at hand. After the publication of *Principia* astronomers and mathematicians undertook the computation of the orbits of these bodies, and Newton himself led the way. Enler made improvements in the methods, and he discovered the remarkable equation which connects the time of describing a parabolic arc with the extreme vadii vectores and the chord. Lambert extended this equation to the ellipse and hyperbola. Leagrange and Laplace gave solutions. Laplace reduces the problem to the solution of an equation of the sixth degree. In 1797 Olbers gave a simple and convenient method, that still keeps its place as the best practical method, having been improved by Gauss, Encke and Oppolzer.

1905, June 2.

THE REQUISITES OF A TRANSIT OBSERVER.

GEO. A. HILL.

FOR POPULAR ASTRONOMY.

The astronomy having for its purpose the determination of the position of heavenly bodies is always in need of good transit observers. It has suggested itself to me that perhaps a few practical hints upon the necessary qualifications for the position would be of interest to readers of this magazine.

A transit observer is one, who, by means of an instrument provided with a reticule of fixed lines in the stellar focus of the telescope, obtains data upon which to base the instant the revolution of the Earth on its axis causes a star, toward which the instrument is pointing, to bisect the lines seen in the field of view. In a nut shell that is what is expected of the observer. It may take him many years to learn how to do it, and then make a failure of the duty.

The number of persons who may claim to have mastered this art is exceedingly small, when compared with those in other professions, and their qualifications in this field will not escape the attention of the directors of astronomical observatories.

First, in considering this subject, we will touch upon what might be termed personal characteristics. A person of a highly nervous disposition will never become a good transit observer. On the contrary he should be one who is not excitable, who avoids constant experimentation in his method of observing, who is satisfied to acquire proper habits of work, and who will secure transits in a cool and alert manner.

He should not permit himself to be hurried, he should always be at the eye end of his telescope in ample time to be comfortably settled on the observing couch before the star enters the field, prepared to grapple with one of the most subtle correlations that exists between the eye and the hand. His vision should be of the best, and as the young observer continues his experience in this particular field, he will soon learn the better his eye sight, the better his work when sifted through the hands of a Boss or Auwers.

Every transit observer is always too early or too late in forming the mental impression of the passage of the star across each of the threads. The one that continually obtains them exactly at the proper time is yet to be found. The young observer will from the start fall into one of two methods of securing transits.

The first is what is called the anticipating method, the other the waiting method, and in these I refer to the time intervals being registered on a chronograph sheet.

The record of the instant of passage of the star across each thread, as seen visually by the observer, is made on the chronographic sheet through the medium of what has been called by astronomers a key. This little apparatus is simply a process by which the operator at any instant breaks the electric circuit existing between the key and a magnet attached to the chronograph, and records the effect of the break through the medium of a pen, filled with ink, moving on the sheet of paper fastened to a large cylinder, on the chronograph.

In the anticipating method the observer starts to tap the key before he actually sees the bisection of the star by the thread, assuming that when the tap is made the star will be on the thread. The second method is one in which the observer does not tap the key until he actually sees the bisection of the star by the thread. In consequence his transits will always be late. The latter process is that now universally adopted by all experienced observers.

In either method, so long as the difference between the true time, and that actually observed, remains constant, and this is the personal equation of the observer, the error of anticipation or waiting is of minor importance, because it can be determined and allowed for, as will be explained later. But if it is not constant then the young observer has a grave difficulty to overcome.

On page 170 of his Catalogue of Fundamental Stars, Professor S. Newcomb discusses this subject, and gives a method he adopted when he was a transit observer. I will quote from that page his remarks.

He says:—"At the same time he (Newcomb) adopted a system which he thinks might possibly, with due care on the part of the observer, nearly eliminate the error in question." (The error to which he refers is the so-called personal equation.) "This consisted in noticing at each tap of the key, whether the sound was simultaneous with the bisection of the star by the thread. If, at the transit of any star, the tap was found to have been too late, an effort was made to make the next tap too early, and this process of equalization was continued throughout the series of threads, so that in the judgment of the observer, the sound of the taps was in the general mean coincident with the bisections."

To the above suggestions my long experience as a transit ob-

server would lead me to demur against recommending such a scheme of observing transits. I will explain my reason. There are two methods of connecting a key in the circuit with the clock and the chronograph. One is called the make and the other the break circuit. In the make system the electric circuit is open from the clock to the chronograph except during the instant a second's mark is being made. The action of the clock causes the terminals of two wires running from it to the chronograph magnet, to touch, and then separate, completing for an instant an electric circuit.

The key in the make circuit system has fastened to it two wires running from the chronograph magnet, and the battery, respectively. One of these wires is fastened to a brass plate on the base of the key, and the other to the spring to which is attached the button which the finger presses downward when a tap is to be made. In the normal position of the key these two points are separated, and their distance apart is regulated by a set screw.

There may be a relatively large or small space between the base point, and the button point, and the touching of the key by the hand is not made manifest on the chronograph sheet until the two points are in actual contact. That was the form of key in use by Professor Newcomb, and the time that elapsed between tapping the key, and making the record on the chronograph sheet depended upon how much play there was between the upper and lower terminals of the two wires.

In modern practice that plan of connecting the key has been abandoned, and the break circuit substituted for it. In the latter, the circuit from clock to chronograph and key is closed, or under the influence of the battery, except at the commencement of each second. At that instant the seconds wheel of the clock raises a very delicate spring, which together with a post against which it rests normally are the two terminals of the wires, and the electric circuit is broken, the effect being transmitted to the chronograph sheet. As soon as the impulse of the clock carries the toothed wheel beyond the spring, the latter instantly drops back against the post, and the circuit is closed. This is the mode of operation for each second, except the sixtieth, which is left off the wheel, so one can tell at what place on the sheet the minute commences.

The same explanation applies to the key for the ends of the wire are in contact in it, but the instant the finger presses downward or makes a tap the circuit is opened, without any regard

whatever to the space through which the button must be pressed before its point touches on the base of the key.

Now to return to the quotation. The human mind is incapable of realizing if a tap of the key was simultaneous with a bisection of a star by a thread. It can only assume a transit was recorded too early or too late, and it is an assumption pure and simple. If it was otherwise, the human mind could be tutored to touch the key exactly at the proper time, and the personal equation of an observer become a phantom.

For a young observer to attempt to make one transit a little late, because the previous one appeared to him a little too early, or vice versa, will inject such a discord into his system of observing as to make his transits worthless.

Place every individual bisection on its own merits, and in the long run your accidental errors will go out, and your systematic error of observing become a quantity that can be depended upon.

In a following paragraph in the work cited, Professor Newcomb nullifies any merit there may be in the plan he has suggested of equalizing transits, by mentioning a peculiarity known to all old transit observers.

It is this. As a star enters the field of view the observer will notice that as the object comes within about its own width of a thread his next impression is that it has jumped across the thread, and left no tangible impression on the mind when it was actually bisected by the thread.

This is a psychological condition over which I have studied a good many years. Of it I have formed this opinion. As a star rapidly approaches the thread the mind is intently engaged in considering the instant when the mental message from the eye to the hand shall be transmitted. When that message is actually carried, and the tap of the key is made, the optic nerve for an instant loses the moving star, and when next impressed on the brain, the object seems to have made an invisible jump across the thread. Every observer who is familiar with this peculiar condition is well aware that just before and after transit a star appears to move faster than it does when in other positions between threads, or appears to leap.

This becomes more pronounced as one continues their tour of observing, and if that is prolonged to four or five hours of steady work, it may become so serious as to affect the results.

Right in this connection there is another interesting subject in an observer's physical make-up that I have noticed in my own experience. On a number of occasions, due to the need to obtain

certain isolated star positions from my regular list, I have observed from the dark in the evening until day light in the morning, not continuously, but I did not secure either rest or sleep in that time.

When a tour of duty has extended over so long a period I have found that between two and three A. M., and I am now speaking of the longer nights of fall and winter, my power of being on the alert to secure good transits has sunk to a low ebb, so much so as to demand a large exertion of will power to carry them on. But what seems peculiar is that after passing that period my mental alertness, without forcing myself, commenced to revive, and from that on to morning I did not need to exert will power to keep me to my work, any more than I did when I commenced in the evening. I do not know if other observers have had a similar experience, but it has been so pronounced in my case that I thought it would be of interest to mention it.

Unless it is absolutely necessary I do not think it good policy to extend a power of observing over more than three hours. The strain on the eyes, as well as on the mental faculties should be abridged below rather than extend beyond that period.

An observer intending to become proficient in the art of securing transits should refrain from using his eyes in physical work on the Sun or the Moon. Constant study of the surface of those two bodies will tend to impair the eye sight for good transit work.

Another important idea to always have in mind is the gradation of light on the threads in the micrometer. The illumination should always be kept to such a faintness that it does not produce a glare, it should be soft, not metallic in appearance, and should be so colored to bring out the best contrast of thread and star.

Some prefer an orange colored illumination, while others may incline in their preference toward a red. My own is toward an orange color, and for this reason. Very few stars will be observed that are markedly red in color; the majority will appear of a yellow tinge, passing into white. A properly flashed orange, or red glass, not too deep in color, seems to my mind to bring out the blackness of the threads, making them appear sharper in outline.

Moreover, one will notice when ordinary spider webs are illuminated with yellow or white light, the edges of the web will, as a rule, appear brighter than the central portion, or to state it differently, they will have attached to them a suggestion of a

fringe of white light somewhat brighter than other parts of the web.

Every transit observer has what has been termed in astronomical nomenclature a personal equation, of which there are two kinds. In the first, as has been explained, he secures his transits either too early or too late, and in the second, as the image of the object is diminished in brightness, the time of its passage across threads is generally recorded by him later than it would have been had it been brighter in magnitude. This condition does not in any manner indicate poor or careless observing, but is due to the stellar images as seen in the telescope. In the case of the bright star we see a large image, very difficult to judge when its center passes a thread, and the trouble enhanced by the bright streamers radiating from the center. In the case of the faint star we have an image approaching more nearly the apparent width of the thread. It has been found from a comparison of transits of a standard observer that those of bright stars are generally recorded earlier than those of faint stars. The method of making this comparison will be mentioned later.

An instrument, called a personal equation machine, has been devised to ascertain the difference between a perfectly true transit, and that recorded by each observer. Briefly, it has the following construction. It consists of a tube, of the form of an ordinary eye piece, in front of which is placed a piece of ground glass on which are ruled lines corresponding to the threads in the micrometer of the ordinary transit instrument.

The piece of ground glass is placed in a frame, and the latter is fastened to clock work so that it may be moved at right angles to the tube corresponding to the eye piece. Behind the frame is placed a total reflection prism, through which a beam of light from a lamp is thrown on to the glass plate. The beam passes through a small hole in another tube just back of the ruled glass, and on looking through the first tube one sees an artifical star.

On the movable plate an electric devise is fastened, and as each line passes the artificial star, the motion being imparted by the clock, the circuit between it and the magnet on the chronograph is broken, and the record made on the sheet. A key, which has just been described, is placed in the circuit with the chronograph, and the personal equation machine. A battery supplies the power for recording the transits.

The observer seats himself before the instrument, and starts the clock. On looking into the artificial eye piece he sees a star

transiting the threads just as he would were the instrument pointed to the heavens. With the key he makes taps as he visually observes each bisection of the star by each thread. The machine records automatically the time when the star was on a thread, and it follows that the difference between the two marks transcribed to the chronograph sheet is the measure of the personal equation of the person making the observations.

The instrument may not exactly record the instant the star was on the thread, when the motion is only in one direction, due to mechanical defects in the adjustment of the points corresponding to the end of each ruled line, but the clock is so constructed that it will drive the plate in opposite directions, and the mean of the two automatic transits thus made will have the error just mentioned eliminated. From a large number of such transits, if the observer has faithfully made them as he would in his ordinary work, he will have his average personal equation.

The second personal equation is obtained with the aid of the transit instrument and the stars. In this investigation the observer must supply himself with a number of wire screens, with a mesh so fine that on placing it before the object glass it will reduce the brightness of a stellar image to any magnitude desired. The screens can be placed in a frame fastened to the telescope, and so arranged as to be placed in front of the object glass, and then quickly removed, or they can be operated through the aid of an assistant.

The actual amount of light cut out by the screen is a matter of simple computation, being based on the number of threads to the inch in the screen, and their diameter.

The observer selects a relatively bright star, and records the transits of the same over half of the threads in his instrument, using the full magnitude of the star. He then quickly places in front of the object glass a screen to reduce the image to any magnitude desired, and records the transits of the reduced image over the remaining threads.

Each thread without the screen, as well as those with it, are by the ordinary processes, reduced to the middle thread, or to the mean of threads. The difference between the average of the first set and the second is a measure of the personal equation of the observer when it depends upon the magnitude of a star. It will generally be found for all observers, that the difference between the transits of the bright star, and the same reduced in brightness by this artificial means, will be an appreciable quan-

tity, with some observers being quite marked, and in general the first will be less than the second.

There are other practical hints that might be of service to an inexperienced observer. Always open your observing house at least an hour before commencing to work. Don't put off until the last minute having your chronograph pen filled and in place, the setting for the first star made in ample time before its transit, and what may appear almost too simple to mention, remove the cap from the object glass. Never be hurried in going from your home to your instrument, with its consequent excitement of mind and body. Active physical exertion or mental excitement just before commencing to secure transits will so change the ordinary personal equation of an observer as to astonish him when confronted with the record at such a time, placed on the equation machine.

Adjust the level bubble so as to keep it of uniform length throughout the year. Place the basis of mercury from which you derive your collimation in one containing sand, and then cotton. It is the best means of keeping down tremors. If the wind comes in around the cracks in the building place over the basin a box seven or eight inches high, or sometimes you will find an ordinary piece of mosquito netting laid over the box will break up the tremors. Make the mercury sluggish by putting in pure tin. In obtaining your stellar image always do it over the mercury, the images that respond to a perfect adjustment by that means by far excells the sharpness that can be obtained from the image of a star.

To become an accurate transit observer is to secure the highest art in astronomical investigations, and while the hints given make no claim to completeness, I am confident they will be found useful.

WASHINGTON, D. C

A Differential Method Generally Applicable to All Solutions Proceeding by Successive Approximations, and Especially Available When These Approximations Practically Fail to Converge.*

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[Communicated by Rear-Admiral C. M. Chester, U. S. N., Superintendent.]

FOR POPULAR ASTRONOMY.

In determining the elements of an elliptic orbit by the method of Gauss, the rapidity of convergence of the successive approximations will depend mainly upon two things—*first*, the rapidity with which the radius vector changes its length; and *second*, the length of the intervals between the three dates of observation. If the eccentricity of the orbit is small, and the observations are only a few days apart, then one or two approximations will generally give the elements with all the accuracy that is desired. When either or both of these conditions are not fulfilled, however, it may frequently happen that a continued repetition of the successive approximations will only involve the computer either in confusion or discouragement: for he will probably find; *either*, that the convergence is altogether too slow to give accurate results after a limited number of repetitions; or, worse, that the successive results are so lacking in concordance or sequence as to indicate an utter failure of the method to yield a final solution. A simple illustration of the latter difficulty is afforded by an attempt to find the root $x = 100$ from the equation $x + 0.008x^2 = 180$, by means of successive approximations. Thus, having reason to believe that the value $x_0 = 101$ is within 1 or 2 per cent. of the truth, we may find $x_1 = 180 - 0.008\,x_0^2$; then $x_2 = 180 - 0.008x_1^2$; and so on. In this manner we find

$$x = 100 \left\{ \begin{array}{ll} x_0 = 101.0 & x_3 = 96.0 \\ x_1 = 98.4 & x_4 = 106.3 \\ x_2 = 102.5 & x_5 = 89.6 \end{array} \right\}$$

showing the uselessness of continuing the approximations on this basis.

These considerations lead naturally to the statement of a definite problem. To make this statement general, we need not confine ourselves to the special problem of determining an elliptical orbit, either by the method of Gauss or otherwise; rather shall we consider *any* problem whatever solved by means of successive ap-

*After this article was prepared for the printer, the writer's attention was directed to the fact that the method herein developed and illustrated has already been given by Oppolzer (Lehrbuch zur Bahnbestimmung der Kometen und Planeten, Vol. 1, pp. 370-371). Feeling that perhaps the several examples given herewith may render the article of sufficient interest to appeal to the younger students of Popular Astronomy, the writer has decided to submit the manuscript in its original form, with no further apology.

proximations. The problem in question may involve either one, two, or any limited number of unknowns. Granting that the results of the first two or three approximations indicate inherent difficulties of the character noted above, the question then arises: Is it possible to find some method by which we can calculate accurate or final values of the several unknowns from the data furnished by the few approximations already performed?

Before presenting a solution of this problem in the general case, it may be well to apply it specifically to one or two simple examples. Thus, let us take the equation cited above, namely—

$$x + 0.008x^2 = 180.$$

We assume $x_0 = 101$, and find

$$x_1 = 180 - 0.008x_0^2 = 98.39,$$

there being reason to know that each of these values is not far from the truth. Then we have

$$dx_1 = -0.016\ x_0 dx_0 = -1.62\ dx_0$$

Now, since x_0 leads to the value x_1, it is clear that if dx_0 denotes a *small* change in x_0, the value $x_0 + dx_0$ must yield the value $x_1 + dx_1$. Hence, if we put

$$x_0 + dx_0 = x_1 + dx_1$$

we provide the condition that an *assumed* value conduces to a second value identical with the first. Therefore $x = x_0 + dx_0$ is the true value of the unknown quantity; or at least is an exceedingly close approximation to it, provided dx_0 comes out sufficiently small. Thus we have

$$dx_0 - dx_1 = x_1 - x_0 = -2.61$$

or, from the value of dx_1 determined above

$$2.62\ dx_0 = -2.61$$
$$\therefore dx_0 = -1.00$$
$$\text{and } x = x_0 + dx_0 = 101 - 1 = 100.$$

In this example we can even apply the method to the values x_4 and x_5 (106.3 and 89.6, respectively) and find $x = 100.1$.

As a second example involving but one unknown quantity, consider the plane triangle ABC, in which C, a and b are given, and the

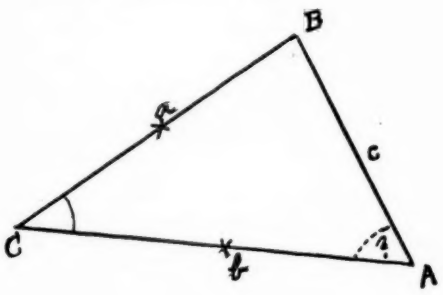


FIG. 1

angle A is required Bearing in mind that in general it is simply the inadequacy of our present methods of analysis which necessitates the application of the method of successive approximations, we may here readily supply a striking analogy to the conditions of our more complicated problems by the assumption that we know nothing of the principles of plane trigonometry except the fundamental law of sines. Thus the following method of finding A by successive approximations naturally suggests itself:

Assume A_0, known to be not far from the true value of A. Then compute $B_0 = 180° - C - A_0$, $\alpha = a \sin C$, $\beta = b \sin C$,

$$c' = \frac{a \sin C}{\sin A_0} = \frac{\alpha}{\sin A_0}, \quad c'' = \frac{b \sin C}{\sin B_0} = \frac{\beta}{\sin B_0},$$

$$\log c_0 = \tfrac{1}{2}(\log c' + \log c'')$$

$$\sin A' = \frac{a \sin C}{c_0} = \frac{\alpha}{c_0}, \quad \sin B' = \frac{b \sin C}{c_0} = \frac{\beta}{c_0}$$

$$A'' = 180^0 - C - B', \text{ and } A_1 = \tfrac{1}{2}(A' + A'')$$

In particular, let us take

$$C = 50°,$$
$$a = 97, \quad b = 103;$$

these values being purposely chosen so that A and B will not differ greatly from each other (in the case of an orbit this condition would find its direct analogy in a small excentricity). In this case a casual inspection of the above formulæ indicates that if A is known within 1° or 2° of the truth, the first calculation will give a very close approximation to the required angle; while a second would undoubtedly give A as accurately as is desired. For, since A + B is constant, we have $\delta B_0 = -\delta A_0$. Hence, as A_0 and B_0 are themselves nearly equal, c′ and c″ must differ from c by small quantities which are opposite in sign and nearly equal. Thus c_0, A′, B′, A″ and A_1 must be very near to the truth.

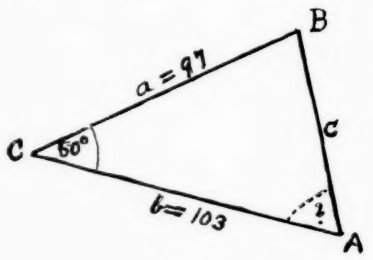


FIG. 2

Now the actual values of A and B are:

$$A = 61^0\ 19'.1; \quad B = 68^0\ 40'.9.$$

Assuming $A_0 = 62^0$, we find

$B_0 = 68°\ 0'$	$A' = 61°\ 24'.5$
$\log c' = 1.92509$	$B' = 68\ \ 48.4$
$\log c'' = 1.92992$	$A'' = 61\ \ 11.6$
$\log c_0 = 1.92750$	$A_1 = 61\ \ 18.1$

A_1 being in error by only 1′.0.

Again, starting with A_1 in place of A_0, we find from a second approximation, $A_2 = 61° 19'.1$, which is the exact value of A.

Let us now take the case in which a and b are quite unequal (corresponding to a large excentricity in the case of an orbit.)

Let $C = 50°$, $a = 50$, $b = 150$. Here our method of approximation will undoubtedly give unsatisfactory results, since our formulæ do not effectively meet the conditions furnished by the given data. As a matter of fact, the true values of the angles are

$$A = 18° 0'.2; \quad B = 111° 59'.8.$$

Now, since A is in the first quadrant and B in the second, the fixed condition that $\delta B_0 = -\delta A_0$ results in the fact that any positive error in the assumed value of A_0 will make both sin A_0 and sin B_0 respectively *greater* than their true values, sin A and sin B. Hence c′ and c″ will both be *less* than the true value c, and thus c_0 will be decidedly in error; consequently the angle B′, being in the vicinity of 90°, and determined from its sine, will probably differ very materially from the truth,—certainly much more so than will the angle A′, whose sine is small. It is thus evident that trouble will arise in the successive applications of the above formulæ to the present example. Indeed, it may be noted in passing that much better results could be expected from a continued repetition of the process, were we to take A′ instead of A_1 as the result of each successive step.

Assuming $A_0 = 18° 30'$, the method gives

$B_0 = 111° 30'$	$A' = 18° 17'.0$
$\log c' = 2.08174$	$B' = 109\ 45.8$
$\log c'' = 2.09166$	$A'' = 20\ 14.2$
$\log c_0 = 2.08670$	$A_1 = 19\ 15.6$

whence it appears that A_1 is more remote from the truth ($A = 18° 0'.2$) then either A_0 or A′. Thus the futility of proceeding further on this basis is manifest.

Let us now apply to this example the differential method which is generally applicable in just such cases. From the foregoing relations we find

$$dB_0 = -dA_0, \quad \frac{dc_0}{c_0} = \tfrac{1}{2}(\cot B_0 - \cot A_0)\, dA_0$$

$$dA' = -\left\{\frac{dc_0}{c_0}\right\} \tan A', \quad dA'' = -dB' = +\left\{\frac{dc_0}{c_0}\right\} \tan B'$$

$$2dA_1 = dA' + dA'' = \frac{dc_0}{c_0}(\tan B' - \tan A')$$

whence, after some slight reductions,

$$dA_1 = -\frac{\sin(B' - A')\sin(B_0 - A_0)}{\sin(B_0 + B')\sin(A_0 + A')}\, dA_0$$

Computing with four place logarithms the value of the trigono-

metrical coefficient, we find $dA_1 = +2.527\ dA_0$. Finally, putting

$$A = A_0 + dA_0 = A_1 + dA_1$$

we have

$$dA_1 - dA_0 = A_0 - A_1 = -45'.6$$
$$\therefore 1.527\ dA_0 = -45'.6,\ dA_0 = -29'.8$$

whence $A = 18^\circ\ 0'.2$, which is the exact value of the required angle.

In order to indicate clearly the application of this differential method to the general problem where n unknown quantities are involved, we shall consider the case in which $n = 3$. Thus, let x, y, z denote the true values of the quantities sought; x_0, y_0, z_0, fairly approximate values assumed at the outset; and suppose that the relations from which the final values are to be found by successive approximations are the following:

$$\lambda_0 = f_1(x_0, y_0, z_0) \quad \mu_0 = f_2(x_0, y_0, z_0, \lambda_0)$$
$$x_1 = F_x(x_0, y_0, z_0, \lambda_0, \mu_0)$$
$$y_1 = F_y(x_0, y_0, z_0, \lambda_0, \mu_0)$$
$$z_1 = F_z(x_0, y_0, z_0, \lambda_0, \mu_0)$$

in which λ_0 and μ_0 are the auxiliary quantities of the solution. From these equations we derive

$$d\lambda_0 = l_1 dx_0 + l_2 dy_0 + l_3 dz_0$$
$$d\mu_0 = m_1 dx_0 + m_2 dy_0 + m_3 dz_0 + m_0 d\lambda_0$$

and

$$dx_1 = a_1 dx_0 + a_2 dy_0 + a_3 dz_0 + \alpha_1 d\lambda_0 + \alpha_2 d\mu_0$$

Eliminating $d\lambda_0$ and $d\mu_0$, we have

$$dx_1 = A_1 dx_0 + A_2 dy_0 + A_3 dz_0$$

and similarly

$$dy_1 = B_1 dx_0 + B_2 dy_0 + B_3 dz_0$$
$$dz_1 = C_1 dx_0 + C_2 dy_0 + C_3 dz_0$$

in which the literal coefficients are determined numerically by differentiation of the given relations and subsequent eliminations. From the manner of deriving these expressions, it follows that if dx_0, dy_0 and dz_0 are any small quantities whatever, arbitrarily chosen, the assumption of the values $x_0 + dx_0$, $y_0 + dy_0$, $z_0 + dz_0$, in in lieu of x_0, y_0, z_0, must lead to the values $x_1 + dx_1$, $y_1 + dy_1$, $z_1 + dz_1$, in place of x_1, y_1, z_1. Hence in the equations

$$x_0 + dx_0 = x_1 + dx_1 = x$$
$$y_0 + dy_0 = y_1 + dy_1 = y$$
$$z_0 + dz_0 = z_1 + dz_1 = z$$

we have the necessary conditions that certain assumed values of the unknowns lead at once to a second set identical with the first. Thus the true values of x, y and z become known. Writing these relations in the form

$$dx_1 - dx_0 = x_0 - x_1,\ dy_1 - dy_0 = y_0 - y_1,\ dz_1 - dz_0 = z_0 - z_1$$

and substituting the preceding values of dx_1, dy_1, and dz_1, we obtain

$$(A_1-1)\,dx_0+A_2dy_0+A_3dz_0=x_0-x_1$$
$$B_1dx_0+(B_2-1)\,dy_0+B_3dz_0=y_0-y_1$$
$$C_1dx_0+C_2dy_0+(C_3-1)dz_0=z_0-z_1$$

from which the values of dx_0, dy_0, dz_0, and hence the quantities x, y, z, may be determined.

As a final example of the process, let it be required to find the values of y and z from the equations.

$$x+y+2z=90$$
$$2x^2+3y+4z=380$$
$$4x+3y-3z=10$$

in which the true values of the unknowns are $x=10$, $y=20$, $z=30$. Let the following form of solution by successive approximations be adopted.

Assume y_0 and z_0; then find

$$x_1=90-y_0-2z_0,\ x_2=\sqrt{\tfrac{1}{2}(380-3y_0-4z_0)},\ x_3=\tfrac{1}{4}(10-3y_0+3z_0)$$
$$x_0=\tfrac{1}{3}(x_1+x_2+x_3),\ y_1=90-x_0-2z_0,\ z_1=\tfrac{1}{3}(4x_0+3y_1-10)$$

In particular we assume

$$y_0=21,\ z_0=32,$$

and thus find

$$x_1=5.00 \qquad x_0=8.49$$
$$x_2=9.72 \qquad y_1=17.51$$
$$x_3=10.75 \qquad z_1=25.50$$

whence a repetition of the process is seen to be impracticable. We therefore proceed as follows:

$$dx_1=-dy_0-2dz_0$$
$$dx_2=-\frac{1}{4x_2}(3dy_0+4dz_0)=-.077dy_0-.103dz_0$$
$$dx_3=-\tfrac{3}{4}dy_0+\tfrac{3}{4}dz_0$$
$$dx_0=+\tfrac{1}{3}(dx_1+dx_2+dx_3)=-0.609dy_0-0.451dz_0$$
$$dy_1=-dx_0-2dz_0=+0.609dy_0-1.549dz_0$$
$$dz_1=+\tfrac{4}{9}dx_0+dy_1=-0.203dy_0-2.150dz_0$$

Since we must put

$$y_0+dy_0=y_1+dy_1$$
$$z_0+dz_0=z_1+dz_1$$

we have

$$dy_0-dy_1=y_1-y_0=-3.49$$
$$dz_0-dz_1=z_1-z_0=-6.50$$

Whence

$$0.391dy_0+1.549dz_0=-3.49$$
$$0.203dy_0+3.150dz_0=-6.50$$

from which we find

$$dy_0=-1.00 \qquad y=y_0+dy_0=20.00$$
$$dz_0=-2.00 \qquad z=z_0+dz_0=30.00$$

which are the exact values of y and z.

PLANETARY ABERRATION.

ROBERDEAU BUCHANAN.

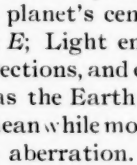
FOR POPULAR ASTRONOMY.

When the writer took up the study of Astronomy, one discussion made a great impression upon him, for he very much disliked it, —that of planetary aberration, which considers the time it takes light to pass from the object glass of a telescope to the eyepiece! The geometrical figure usually appended looks all right where the length of the telescope appears to be one-fourth or one-fifth of the distance of the planet and which ought therefore to give very accurate results. This method is sanctioned by Chauvenet, Brunnow and other eminent writers.

It is strange however, that by supposing the aberration to be computed for the surface of the Earth, the fact is overlooked that it would be in error, not only by the amount of parallax, but by the distance the light is supposed to travel.

The problem is simply to find the angle of the resultant of two forces, and the formula may be correctly derived as follows:

Let P be the position of the planet's center while the Earth's center is at E; Light emanates from the planet in all directions, and one ray reaches the point F just as the Earth arrives there, the planet having meanwhile moved to Q. The angle $P\,F\,Q$ is the aberration. We see the planet in the apparent direction $F\,P$, while its true direction is $F\,Q$ at the same instant. Hence the sign of the aberration is negative.



There being no lines in the celestial sphere none are needed in this figure, and moreover the positions of the letters are immaterial. When the planet is stationary $P\,Q\,F$ form a right line the planet is moving towards the earth, and the aberration is zero. At retrograde motion the planet moves from Q to P which makes the aberration positive. The planet soon becomes stationary again moving from the Earth, $F\,P\,Q$ from a right line and the aberration is again zero.

$P\,E$, $P\,F$ and $Q\,F$ represent the planet's distance from the Earth, they are considered to be equal since they are very great compared with $P\,Q$. We know the time it takes for light to travel from P to F and knowing the velocity of the planet we can compute the angular space $P\,Q$.

Light required 497.78 seconds to pass from the Sun to the Earth at its mean distance; it will pass over the distance $PF=\rho$ in

497.78 ρ seconds, or dividing by 86400 the time in decimals of one day will be

$$\frac{497.78\rho}{86400}$$

We are supposed to have the true right ascension and declination of the planet computed for equal intervals of n days, where n may be taken as one day for Mercury or 8 days for Neptune. If now Δ_1 represents the first differences of the planet's right ascension or declination $\frac{\Delta_1}{n}$ will be the motion in one day. It will be more accurate however to take the mean of the two differences above and below the computed date which we will represent by $\frac{\Sigma\Delta_1}{2}$. This should generally be corrected for third differences; and if the interval of the computation is much greater than n given above the correction for fifth differences may be necessary, therefore we have the motion of the planet in one day, in seconds of arc

$$\frac{1}{2n}(\Sigma\Delta_1 - \frac{1}{6}\Sigma\Delta_3 + \frac{1}{30}\Sigma\Delta_5)$$

And the aberration,—the angular distance $P\,Q$,—in seconds of arc is the product of these two quantities; giving the numerical portion by its logarithm.

$$\text{Aberration} = -\frac{[7.45949]}{n}\ \rho\,(\Sigma\Delta_1 - \frac{1}{6}\ \Sigma\Delta_3)$$

which is applicable either to right ascension or declination.

The formula given in *Popular Astronomy*, vol. IV, p. 36, depending upon hourly motions is wrong both in theory and practice; in practice because the hourly motions have not been computed at this stage of the work; and in theory because the hourly motions include, besides the motions of the planet itself, the change in the aberration, which latter added will give a wrong result.

Professor Chauvenet in deriving the formula for the value in one hour is also likely to mislead the computer for the same reason. If the interval of the computation is not greater than the values of n mentioned above, four place logarithms will be sufficient for the right ascension of every planet; four for the declination of Mercury, and three for the declinations of other planets.

VARIABILITY AND THE ASTEROIDS.

GRACE AGNES THOMPSON.

For Popular Astronomy.

The recent announcement by Harvard College Observatory of the establishment by Professor Oliver C. Wendell of a variation in the light of the Asteroid Eunomia, is attracting new attention to a subject that was already most interesting. It is the third case of variability that has been discovered among this group of minor planets. The study of the variable stars, from the very perversity of these objects in evading all certain conformity to any known theory of cause and effect, has been always intensely fascinating to astronomers. But only during the past twenty-five years, since astronomical photography has become efficient, so that a comparison of such stars from different photographs could be made, has any definite knowledge of their puzzling phenomena been gained, by which the various types of variation could be classified.

At Harvard Observatory, where nearly 200,000 photographic plates, mapping the sky from pole to pole and giving a consecutive history of the entire steller universe for more than sixteen years, are kept for reference, such study has been especially possible. And it is here that by far the greater number of known variables have been discovered. A special detailed examination of photographs of the nebulous regions of the sky for the purpose of detecting variable stars was begun at Harvard early last year, with the result that more than 1300 such stars have been added to the 1500 previously known, of which latter list 970 were also found by observers at Cambridge since the year 1890.

The study of variable stars has therefore all the freshness and impulse of a much newer subject. Curious and interesting, however, as these inconstant objects are, and important as is the study of them to science, the subject receives a twofold attraction when it is known that not only the distant stars, but some of the bodies that form our own system—some of the planets themselves,—are subject to periodic changes in brightness. The Asteroids, it should be remembered, are the smallest planets in our system, and were unknown till the discovery of Ceres in 1801. Up to that time astronomers had been accustomed to wonder greatly at the very considerable gap which they had noticed between the orbits of Mars and Jupiter, and which was quite remarkable when they remembered the regular progression of the distances between the

other planets. Several more of the larger of these minor planets were discovered during the first half of the nineteenth century, but the introduction of larger and more powerful telescopes after 1850 quickly made surprising additions to the list. It is difficult to realize that individual spheres so infinitesimal when compared with even our small Earth are traveling like it in a regular course around the Sun, eaeh separated by leagues of space from any other body, and each held in its orbit by the same mighty forces that govern the greatest of the planets. Think of an Earth only 100 miles in circumference! Yet many of the Asteroids are even less. The largest of the group is hardly more than 400 miles in diameter. These *globi parvuli* number more than 500, and they circulate outside the orbit of Mars at distances ranging from 2—400,000,-000 miles from the Sun, the periods of their revolution occupying from three to nine times the length of an Earth year.

Just what was the origin of these Asteroids, it is impossible to state. An old hypothesis started by Olbers that they are fragments of one large planet was destroyed years ago. Nothing has yet been determined with regard to their formation, except that they have all essentially the same character. Possibly a further study of variation in light as established in the three Asteroids, Eros, Iris, and Eunomia, will reveal some information on this point. One fact in regard to their orbital motion is worthy of mention. Instead of traveling in regular orbits, each concentric to the others like the rest of the "wanderers," the paths of many of the Asteroids are curiously interlaced, so that these bodies cross and intercross each other, without, however, as has been certainly ascertained, the least danger that there will ever be any collision between them.

The discovery by Dr. Oppolzer in 1901 that the light of Eros is variable suggested some photometric problems of great interest to the science of astronomy. "If, as seems probable, we assume that the variation is due to the rotation of the planet," Prof. Edward C. Pickering, Director of Harvard Observatory, said at the time, "we can, from measures of its light determine the time of rotation, and the direction in space of the axis of rotation. Owing to the varying position of the observer with regard to the planet, much information can be obtained which is impossible in the case of a variable star."

Since that time a special study of this Asteroid has been made at Harvard Observatory, with numerous careful observations whenever the planet was in favorable position in the sky. Eros, on account of its faintness, is often a difficult object to observe satis-

factorily and requires constant watching. Its rapid motion also renders it difficult to compare the observations on different nights, without using different, and in some cases distant, comparison stars. Fortunately, however, the change in light is so rapid that consecutive observations of a large part of the light curve can generally be made. The opposition of 1894 would have been particularly favorable for these studies, since the planet was then very near the Earth, but it had not at that time been discovered and special observations were therefore not made. The photographs that were taken during 1893 and 1894 fail to show any marked variations in light, from which fact it seems probable that the range was at that time small. The plates taken during 1896 give more conclusive evidence of changes, while the photometric measures made at Harvard Observatory in 1898 furnish an accurate determination of the times of maximum, and of the range for that epoch.

These researches led, during the latter part of 1903, to the discovery that Iris is also variable, and new interest was awakened, for it was speedily found that the discussion of any conditions that had seemed peculiar to Eros was applicable to Iris as well. Besides, here was a second and brighter planet, which offered the same manner of opportunities for solving the problems mentioned above. A series of measurements of the light of Iris were made by Professor Wendell with the polarizing photometer, with sliding achromatic prisms, attached to the 15-inch equatorial telescope of Harvard Observatory, with the result that a variation was established like that of Eros, having a period of about six hours, thirteen minutes. Moreover, Iris is bright enough to be readily observed during a large part of the time, though unfortunately the change in light is frequently so small that it can be determined only by observations in which the accidental errors are extremely small. In fact, the observations of Iris made at Potsdam in 1884 fail to show this variation, either because the range was then too small, the period was then different, or the errors of observation rendered the variation imperceptible. It is due to one or all these eauses, no doubt, to the latter in particular, that the fluctuations in light of these Asteroids were not discovered many years ago; for the range is only two or three tenths of a magnitude, and the variation would have been uncertain, even at the time of discovery, but for the very small accidental errors which occur in observations made in the way just described.

In order to render the examination of these objects suitable for

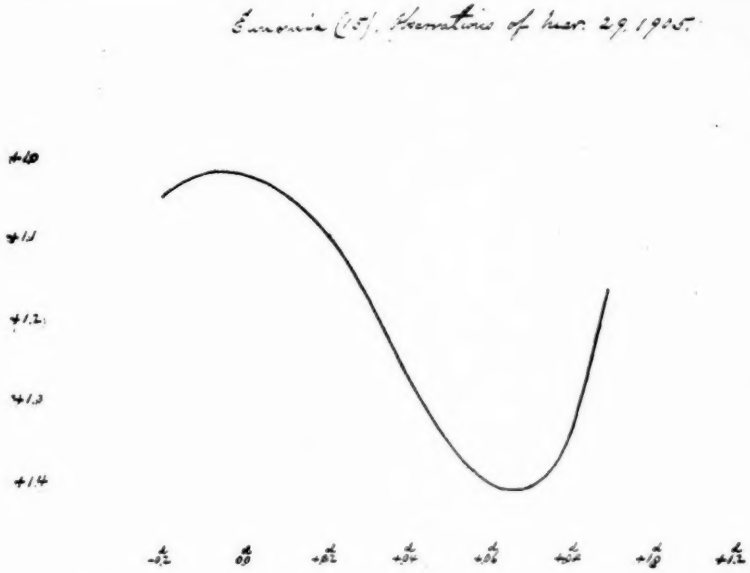
photometric study, it will be found necessary to apply four corrections to the observations. First, for the velocity of light; second, for the distance of the Sun and Earth; third, for phase; and fourth, for the direction in space of the axis of rotation. For, if this axis were pointed toward the observer, no variation would be perceptible, while the range in brightness would attain its maximum value when the axis was at right angles to the line of sight. Neither of these conditions can be fulfilled exactly, since the position of the axis is probably fixed, and the inclination of the orbits of Eros or of Iris and the Earth would make great changes in this angle.

Assuming, then, that the variation in light of these planets is due to their rotation, two explanations may be offered as in the case of variable stars of short period. First, that they are darker on one side than on the other, as is probably the case with Iapetus, the outer satellite of Saturn, and secondly, that they are elongated, or double, as has been assumed by M. André and others. In the first case, the successive maxima would always have the same intensity, and would succeed each other at equal intervals which would be equal to the period of revolution. The same would be true for the minima. In the second case, if the two bodies differed in diameter, the successive maxima and minima might have unequal intensities, and if the orbit was elliptical the intervals between them would be alternately long and short. This seems to be the case with both Eros and Iris, and the first hypothesis seems therefore improbable.

On the other hand, if the variation in light is caused by two similar bodies alternately eclipsing each other, it is difficult to see how more than half the light can be cut off in each case, and the minima more than three-quarters of a magnitude fainter than the maxima. It then becomes necessary to assume that the two bodies are of unequal brightness, that they are elongated, or that we have in each planet a single body of the shape of a dumb-bell. Some observers have found the minima two magnitudes fainter than the maxima. To account for this, we should be obliged to assume that one axis of the body was six times as long as that at right angles to it. Observations show that the light of both planets is continually varying, while if the case were that of a simple eclipse, as in the stars of the Algol type, we should expect that it would retain its full brightness for a large portion of the time. If the bodies were of the same size, and the orbit circular, it might be impossible, from the light curve, to distinguish between the two hypotheses. The fourth of the corrections mentioned above, how-

ever, furnishes a means of distinguishing between them in any case. If the body is dark on one side, the time of revolution will equal the interval between the successive maxima, and the correction for the position of the observer will be proportional to this quantity. If then the position changes 180°, the correction will be one-half the interval between the successive maxima. In the second case, the time of revolution will be double this, that is, equal to the interval between a given maximum and the next but one, so that the correction for position will now be twice as great as before, and approximately equal to the interval between the successive maxima.

Much material already exists for determining the constants mentioned above. Several of the photographs of Eros taken in 1893, 1894, and 1896, had an exposure of an hour or more. Owing to the motion of Eros, it formed a trail on each of these plates, which in some cases shows distinct variations in brightness, — a fact which was noticed when the plates were first examined, but was then supposed to be due to changes in the haziness of the air. As this is an easy method of discovering the variability of an Asteroid, it is hoped that astronomers engaged in a photographic search for such objects will examine carefully all trails, to detect any changes in intensity. An examination of forty-one Asteroid trails photographed in 1903 with the Bruce telescope, seven of them on a single plate, failed to show, except in one or two instances, any change beyond that apparently due to varying atmospheric absorption. Generally, more than one Asteroid appeared on each plate, and in such cases all showed the same changes in intensity. The variation in light of Eunomia, — the third Asteroid whose fluctuations have been proved, — was established by Professor Wendell in the same manner as that of Iris, the observations being begun March 15, 1905. Eunomia was then near its second stationary point, so that it could be compared with the same star, $+13°$ 1875, magnitude 9.0, until April 1, 1905. The period, $0^d.1267$, does not differ greatly from that of Iris, $0^d.1295$, and in both cases it is still doubtful whether this period should be doubled. A careful series of observations by Mr. Leon Campbell, using Argelander's method, seems to indicate that in the case of Eunomia the period should perhaps be doubled, and that the intervals between the successive minima are alternately long and short. It is, however, difficult to see on theoretical grounds how this can be the case, or how the alternate maxima and minima can differ in intensity, if the variation in light is wholly due to the unsymmetrical form of the body. But as yet no explanation can be



offered that will satisfy all conditions of the problem or establish the theory.

It is unfortunate that no easy means exists for observing and following known Asteroids, except when near opposition. But every advantage is taken of these more favorable attitudes of our curious little sister planets. Eros, the first to be found inconstant, has naturally received world-wide attention. A very large number of photometric measures of this planet have been made here since July, 1900. Excellent light curves were obtained during 1903 by Professor Bailey with the 13-inch Boyden Telescope at the southern station of Harvard Observatory at Arequipa, Peru, although Eros was fainter than the twelfth magnitude. Unfortunately, the maximum brightness of the planet did not increase during 1904 above this magnitude, nor does it throughout the present year, which fact renders the opposition of 1905, on August 7, one of the most unfavorable that can occur. The most important observations that can be made during this opposition are those of the variation in light, both visually and photographically. The planet's position in the sky during 1905, will be, in general, nearly opposite to that it had in the spring of 1901, when its variability was discovered.

Not only Eros, but Iris and Eunomia also, are being zealously watched. And it seems probable that valuable observations may be obtained during the next few months, with the numerous large reflectors and refractors available, regardless of the fact that all other conditions are not especially propitious.

THE ATMOSPHERE.

O J. B. CORDEIRO.

In a previous article (POPULAR ASTRONOMY, January, 1905,) the writer has called attention to an upper limit of the atmosphere based upon his theory that all matter less dense than the universal ether is repelled by heavy bodies. According to this view, at the level where this density of the atmosphere is reached, which is not very far from the Earth's surface, the atmosphere will be continually driven off, and we can therefore speak strictly of this level as the upper limit. In the article referred to, this height is given as "something like 210 miles," which was copied from another writer who does not state how he derived it.

In *Ganot's Physics*, we find: "Meteorites have been seen at a height of 200 miles and as their luminosity is undoubtedly due to friction against the air, there must be air at such a height. This higher estimate is supported by observations made at Rio Janeiro on the twilight arcs by M. Liais, who estimated the height of the atmosphere at between 198 and 212 miles. The question as to the exact height of the atmosphere must therefore be considered as awaiting settlement."

Prof. Newcomb states (*Universal Cyclopædia*): "But observations on meteors show that it (the atmosphere) really extends to a height of at least 100 miles, and indeed at that height is sufficiently dense to cause rapid combustion of a meteor passing through it. Observations on the Aurora lead at least to a suspicion that this phenomenon sometimes takes place at a height of three, four or 500 miles. We could hardly suppose it to occur in an absolute vacuum, though it would be unsafe to infer from this that the medium in which it occurs is an extension of the atmosphere proper."

Let us calculate this height by the barometrical formula. This formula (see *Barometrical Determination of Heights*, Lyon & Chamberlain, N. Y.,) is $h = k \log \frac{D_0}{D_x}$ (d) where D_0 is the density at the lower limit and D_x the density at the upper limit. $\frac{1}{k}$ is the

logarithm of the ratio of the increment of density for a unit of height to the density.

For simplicity we shall suppose that the temperature of a column of air is uniform throughout and that of freezing, and that the force of gravity remains the same throughout. Taking the density of the ether as 10^{-17} times that of water (Kelvin's determination), and that of air as .0012932 (Regnault's determination), we have $h = k \log 1.2932 \times 10^{14}$. Taking a meter as the unit of height we have $k = 18416 \times \frac{1}{M}$ where M is the modulus of common logarithms.

$\therefore h = 18416 \times (14.1116813)$ which gives us 259,880 meters at the height where the density of the atmosphere becomes equal to that of the ether. This is equivalent to 161 statute miles. Now the force of gravity does not remain constant throughout this height, but becomes about $\frac{1}{20}$ less at a height of 100 miles. Therefore the effect of neglecting this would be to make the calculated height somewhat less than the actual height. On the other hand, if the average temperature throughout the column is less than freezing, then our calculated height is somewhat greater than the actual height. These two factors tend to counterbalance each other and therefore make our error less in neglecting them. We do not know what the temperatures are at high levels, but they are probably considerably below freezing. On the whole, therefore, we may conclude that the height we are seeking is not very far from 160 miles, and we shall assume this as the height of the atmosphere.

From the barometrical formula we have

$$\log D_x = \log D_o - \frac{h}{k}. \quad \therefore D_x = \frac{D_o}{e^{\frac{h}{k}}}$$

Now let the mass of a unit in one column, say a parallelopipedon of air with a base of one square decimeter and a height of one meter be dm:

$$\therefore dm = \frac{12.932}{e^{\frac{h}{k}}} .dh = D_x.dh$$

the total mass will be $\int_o^h \frac{12.932}{e^{\frac{h}{k}}} .dh$

which is $\Big|_o^h 12.932 \cdot ke^{\frac{-h}{k}}$

For the upper limit this quantity is negligible. We have therefore

$M=12.932\times7998$. (2). This gives us as the mass over a sq. decimeter of the earth's surface 103.432 kilos which is about the standard pressure at 0°, sea level. We can easily from (2) find the level of half mass, or the level where there is as much air above as below. This gives us a height of about 17,400 ft., so that half the air lies very close to the surface.

The pressure is proportional to the density, so that we have for the pressure per square decimeter at the upper limit approximately, $p_h=10^{-12}$ kilos.

Now the air at the upper limit may be considered as subjected to a pressure p_h on one side and nothing on the other, as gravity at this point does not come into play, but on the contrary, according to our theory, it is driven off with great velocity on crossing this limit. It will be interesting to compute the rate of this loss.

$$\text{The acceleration} = \frac{dv}{dt} = \frac{p_h}{m} = \frac{10^{-12}}{10^{-17}} = 10^5.$$

$\therefore$ $v=10^5$ decimeters per second or 104 meters per second.

Now although the repellant force of gravity may be small, on such extremely tenuous matter it will impart almost instantaneously an extremely high velocity so that we may suppose that the air which has crossed our upper limit offers no resistance to the somewhat rapid velocity of expansion at this level as calculated above.

A layer of air 10000 meters in thickness will thus be lost every second and as this air possesses a density 10^{-17} that of water, the mass lost every second from each square meter of surface will be 10^{-10} kilos.

Now if we take about 5536×10^{10} square meters as the surface of our upper limit we find that the total mass of air lost per second is 5536 kilos. If we take in round numbers the total amount of air attached to the earth as something like 5500 million million tons, the amount lost per second of 5536 kilos is insignificant. This loss in a day would amount to something like 478,310 tons of air or 174,583,000 tons in a year.

The process is so slow that it would take something like 30,000,000 years for a complete dissipation of the atmosphere. When the attraction is less, as in a body like the moon, the time of dissipation would be greatly decreased, and this would explain why the moon has long since lost her atmosphere.

SPECTROSCOPIC WORK FOR CLASSES IN ASTRONOMY.

SARAH. F. WHITING.

FOR POPULAR ASTRONOMY.

The intelligent discussion of the problems of Sidereal Astronomy is impossible without a knowledge of the fundamental principles of spectrum analysis. The later refinements of the subject, which enter into the interpretation of "peculiar spectra" and also the "Doppler principle" may wait in the interest of clear ideas for full statement and illustration until near the end of a course of lectures. During the winter months however, where observation through the telescope is less practical, laboratory work may well be planned which will make the future discussion of the condition of the stars, not a mere "cram" of statements which the mind fails to apprehend, but a most valuable training in the scientific method.

At Wellesley College one year of Physics is pre-requisite to the course in Physical Astronomy, but it is found that few students have had more than a glance at a spectrum. Moreover the interest in any piece of work is greatly increased when it is to find immediate practical application.

Three conclusions, now almost axiomatic, lay behind Secchi's marvelously successful "lumping" of the stars into four types, and the later elaborate classification of the Draper catalogues involves no others. It should be the object then of series of exercises to make clear these inductions, to show how work in the laboratory with things we can handle must accompany observation with the telescope, and incidentally to make as familiar as A B C the common designation of lines, whether by Fraünhofer letters, by Kirchhoffs number, or by wave lengths.

Students at this early stage need much help to reason from the seen to the unseen, from what can be taken in with the natural eye to what can only be apprehended with the "eye of the mind," to get a mental conception of the mechanism which gives rise to phenomena. The clear, patient, enthusiastic work of the lecturer and demonstrator is here as everywhere the first essential of success.

When the student sees that the light given out by a solid heated to incandescence, as the carbon points of the electric lantern, on analysis, shows a complete band of colors, his scientific imagination should conceive of the fractions of atoms, the electrons, highly excited by heat but under great constraint because

"in a crowd" causing complicated periodic disturbances in the ether of every wave length. He should also be able to reverse the principle, and when he sees the exquisite ribbon of colors which is the background of the spectrum of Sirius recognize, light from a photosphere of the nature of a solid.

When the student sees on analysis that the light given out by any incandescent gas or volatilized substance consists of lines of color here and there, the imagination must conceive of vibrating electrons, with room enough to vibrate in their characteristic way, thus causing a harmonic series of ether waves to radiate from them as sound waves from a bell or string.

When the gorgeous spectrum from the electric arc is seen crossed by a dark line in the yellow, because the vapor of sodium has been interposed, the imagination must conceive of light given out by electrons closely packed so that all sorts of waves are present, passed through a medium containing sodium electrons capable of swinging by "sympathy" at certain rates. Such light when analyzed, will show by lacunae or gaps what substances were present in the absorbing atmosphere. The student should reverse the principle learned in the laboratory, and when he sees the spectrum of Sirius crossed by dark lines in the precise place of the lines of hydrogen say:—hydrogen exists in the atmosphere of Sirius.

Lecture room experiments with the electric light are a desirable preliminary, also experiments with forks and strings showing the sound analogies, but laboratory work by the student is essential. Faraday himself said he never could understand an experiment until he had done it.

Our series of exercises, besides illustrating the principles above noticed, seeks to show the difference between the normal and prism spectrum the effect of greater dispression in resolving lines, the effect of greater heat or electrical excitation in changing the character of spectra, and also to make it possible to know approximately where a line of the spectrum is when it is spoken of by its wave length.

The apparatus at our disposal for this work for over a hundred students is two direct vision spectroscopes*, three one prism

* The large direct vision spectroscope of Browning, of London, with careful adjustment will show the D line double the Miniature spectroscope of the same make is a gem, and most of the work which follows can be done with it alone. The little French direct vision spectroscopes are cheaper but far inferior in dispersion and definition.

Since students are to see in the laboratory the spectra of the gases and metals, I have lately used for lecture illustration a device of Professor E. C. Pickering

spectrometers, a spectroscope giving the dispersion of several prisms, an induction coil with Plüker tubes and accessories, spectrum maps†, Ives and Wallace replicas of gratings.

The following is the list of exercises found practicable.

1. A large direct vision spectroscope adjusted on a support where it receives light from the sky is used to show the lines from C to G.

2. A second direct vision spectroscope, with slit wide open, is used to show the absorption of such liquids as copper sulphate, purpureum, didymium. These two exercises show the analogous action of absorbing atmospheres whether millions of miles away or immediately behind the slit.

3. Bright line spectra are illustrated by the observation with a one prism spectroscope of the familiar list of light metals volatilized in the Bunsen flame.

4. A wide dispersion spectroscope,—the same one which belongs to the telescope for the observation of solar prominences, is used to show how the sodium line can be widely resolved. Rowland's photograph of the region of this line is shown in connection, and the mind is led from the simple spectrum to the complicated one of more powerful spectroscopes.

5. The miniature direct vision spectroscope of Browning, with its reflecting prism in place, is directed at the sodium flame while light from a high window is admitted at the side. The sodium line and the D line are seen in the same position. Sometimes the two hydrogen lines are shown to be coincident with the C and F lines in the same way.

6. In a dark laboratory Hydrogen, Helium, Nitrogen, Argon,

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which it may be suggestive to some if I detail here.

Arrange to receive the spectrum from the lantern or Sun upon a screen of black velveteen considerably larger than the spectrum and stretched upon a frame. Prepare lengths of coarse black foundation lace the width of the spectrum, and paste upon them strips of glazed white paper (a kind of blotting paper is best) of the width proportional to the intensity of the line, and at intervals proportional to the wave length. These strips can be hung by bull-dog hooks on a wire a few inches in front of the velvet screen, in such positions that each line is in its appropriate color. The effect is surprisingly fine. Sodium and Helium one above the other are on one of my strips, the red and blue spectra of argon on another, the entire series of Hydrogen lines on another. Only four of the lines of Hydrogen are illuminated, but the rest illustrate the photographic spectrum.

† The colored maps of Kirchhoff and Bunsen and of Huggins and Miller, found in most laboratories, are of great use in preparing the student for observation, and for indentification of lines. They also serve as guides for the simple plots of spectra required.

and possibly some other gases and metals are rendered incandescent by the induction spark The nitrogen is shown as the most convenient illustration of a complicated vibration which causes a banded spectrum. The argon is shown first without a condenser in the circuit, and then with the condenser. The quick transition from the red to the blue spectrum shows how velocity of movement increases with temperature so that the point of maximum intensity shifts towards the violet.

7. A pasteboard tube closed, with the exception of a slit at the further end, is directed towards the light of the sky, while a grating ruled on glass or a replica is held next the eye. The relative dispersion of the blue in this normal spectrum can be compared with that of the prism spectrum.

8. The classic maps of the spectrum, those of Fraünhofer, from Schellin's spectrum analysis, of Kirchhoff, Aengstrom, the Draper photograph, the map of Cornu, of Langley and finally the Rowland strips are displayed for study, and a table is made, of the principle lines, with Kirchhoff's number, Aengstrom's wave length, and finally the standard length in tenth meters. This is desirable because in the literature of the spectroscope as applied to Astronomy all designations are encountered.

The students notebook work consists in careful descriptions of the apparatus employed accompanied by drawings, in detailed record of observations, and spectrum maps finished with colored crayons after the order of the elementary maps on the Kirchhoff charts. These are plotted to a convenient scale from tables furnished upon cards. To show whether the application of all this to the subject in hand is apprehended, notebook answers to a list of questions on the interpretation of photographed spectra of the envelopes of the Sun and stars are advised.

In the Spring and Summer months on sunny afternoons many see the superb spectrum obtained with a six-foot concave grating and mark the hundreds of significant absorption lines in the light of our nearest star as they troop by the eye from the red to the blue. Also at a late day in the year glass contact prints from plates of the Bruce telescope, selected by Mrs. Fleming as interesting fields, are mounted on replicas of the pieces of apparatus used at Harvard Observatory obtained by permission of Professor Pickering. The plates are studied in the ordinary manner, records being made of the coördinates and the type of each star spectrum. Such laboratory work followed by the study of a set of lantern slides made from the Tulse Hill Observatory

Annals, showing in its broad outline the present state of the discussion as to the evolutionary sequence of the stars, enables the students not in utter bewilderment but with informed and reverent intelligence to join in the L' Envoy of Sir William and Lady Huggins;—

"We conclude filled with a sense of wonder at the greatness of the human intellect, which from the impact of waves of ether upon one sense-organ can learn so much of the universe outside our earth; but the wonder passes into awe before the unimaginable magnitudes of Time, of Space, and of Matter of this universe, as if a voice were heard saying to man, 'Thou art no Atlas for so great a weight'."

Whitin Observatory.
Wellesley College.

CHART OF FAINT STARS VISIBLE AT THE LOWELL OBSERVATORY.*

PERCIVAL LOWELL.

For Popular Astronomy.

In going over Tucker's Charts of Faint Stars for Magnitude Comparison (Publications of the Astronomical Society of the Pacific, Number 37) with a view to photographic applications Mr. Lampland found that the faintest stars on the Lick chart were perfectly visible here though the objective is 24 inches aperture instead of 36 inches. To test the point thoroughly we selected one of Professor Tucker's richest fields, the one following δ Ophiuchi, and separately charted it. Considering the times at which we were able to work, for the rainy season and moonlight interfered with us, our two charts Mr. Lampland's and mine agree with one another both in the number of stars observed and the positions which these ocupy.

More stars proved to be visible here than were recorded in the Lick Chart, on the proportion of 173 to 161 such being the whole number mapped in the Lick Chart and my own respectively. The two sets were made up in magnitudes as follows:

	Lick Observatory Chart	Flagstaff
8-9	1	1
10-11	5	5
12-13	19	19
14-15	42	46
16-17	94	102
	161	173

* See Frontispiece.

The Lick chart was based upon earlier ones made at Harvard with the 15 inch Merz and at Washington with the Naval Observatory's 26 inch Clark glass—Professor Tucker in the article referred to above gives the number of stars in the Washington chart, omitting those not seen by him, and the number he added as

original (Washington)	new (Lick)	
61	100	= 161

Passing the same course we have as between the Lick and Flagstaff, fifteen stars down on the former chart not being seen

original (Lick)	new (Flagstaff)	
146	27	= 173

He does not state the number of stars he found absent from the Washington chart, but applying the same ratio that he did

	Washington	Lick	Flagstaff,
	65	161	173

The present chart is by no means definitive of what may be charted here as moonlight and the rainy season were both drawbacks but it may serve for comparison with those of elsewhere.

THE LEVEL OF SUN-SPOTS.

WALTER M. MITCHELL, PH. D.

FOR POPULAR ASTRONOMY.

There is probably no heavenly body with which we are so familiar in every day life, as that one which supplies us with light and heat,—the Sun; and of all the phenomena of the Sun, visible to those provided with only small instruments the sunspot is the most easily observed and the most interesting. To the inevitable question "What is a sun-spot?" astronomers are obliged to reply that it can not be conclusively answered at present. A distinguished astronomer, has recently stated, "That the sun-spots are the most extensively studied and least understood of all solar phenomena." This is unfortunately only too true; altho through the many observations which have been made, both visually and photographically, our actual knowledge of the true state of the Sun has greatly increased since the days of Scheiner and Galileo.

If then we can not say *what* the sun-spots are, let us approach the subject from another point of view and ask *where* the sunspots are, meaning the situation of the spot in the general solar make-up. Thus it will be the purpose of this paper to sketch

briefly some of the investigations which have been made with the idea of determining the level of sun-spots.

In questions concerning "level" it is necessary to have some fundamental plane or surface of reference. For the Sun we are most naturally led to look upon what is known as the "photosphere" as satisfying this condition. But there is an objection to this for a theory of the Sun proposed by Schmidt in 1891 considers the photosphere to be not a reality, but a kind of optical illusion. His theory supposes the Sun to be a ball of glowing gas, the density increasing towards the center, the photosphere as we see it being the effect of refraction caused by the action of layers of vapors of different densities upon the path of a beam of light. The prominences, faculæ, spots, etc., are explained as the effects of irregular refractions due to local changes of density within the gas ball.

This theory worked out as it is with careful mathematical analysis is interesting and valuable, but few observers of the Sun will be content to believe that the phenomena that they see are illusions and not realities. The perspective appearances of spots and faculæ as they approach the limb, the displacement of lines in the solar spectrum due to the Sun's rotation, and the existence of a reversing layer are not satisfactorily accounted for by this theory. Also the spectrum of such a body, as shown by Scheiner, would have the absorption lines very much broadened, while in reality the lines in the solar spectrum, with a few exceptions, are fine and narrow. Moreover it seems hardly possible that the outer portions of a gas ball as supposed by Schmidt, exposed to the cold of space, could maintain their gaseous state without condensation, and hence a cloud photosphere of the accepted kind would necessarily exist after all.

Returning then from this digression it will be maintained that the photosphere does exist as a stratum most probably formed by the condensation of the gases of elements of high boiling point, and it will be employed as the surface of reference.

The methods of determining the level of sun-spots can be divided into two classes, the direct and the indirect. The first includes the purely visual observations. The second includes the results derived by spectroscopic observations, by the rotation periods of the different solar layers, and by investigations of thermal radiation.

Visual. In visual observations for ascertaining the level of sun-spots, the general method consists in determining the widths

of the penumbra on the near and far sides of the spot as it approaches the Sun's limb. The penumbra on the side farthest from the limb is usually narrower than that on the other side, thus the spot has the appearance of a cavity or saucer-like depression.

The theory that the sun-spots were depressions of this character in the photosphere dates its origin to Wilson of Glasgow. He noticed the characteristic perspective effects that are caused by depression, in a well developed spot in 1769, and the saucer-like conformation of sun-spots was universally admitted for over a hundred years, notwithstanding occasional failures on the part of the spot to conform to the required conditions. However this was not to last, for at a meeting of the Royal Astronomical Society in December 1894, F. Howlett presented to the society several volumes of drawings of sun-spots extending over a period of about 35 years. The drawings were made with the purpose of testing Wilson's theory. Howlett's conclusion is that his visual evidence absolutely refutes the depression hypothesis, "that instead of the penumbra of a spot possessing *shelving sides*, sloping down towards the umbra, the penumbra presents a *convex* surface; that is to say a curve conformable to a general contour of the solar orb."

In 1895 Sidgreaves of the Stonyhurst Observatory published the results of an examination of over 2000 spot drawings. Of 187 single tests of spots suitably situated for the purpose, he finds 140 opposed to the depression theory, and but 47 which might seem to confirm it, he adds, "But if we estimate the worth of 47 cases quoted for Wilson on the lines of his own valuation of their testimony, nearly all the 47 witnesses will go out of court."

It may be added that the careful study of a large number of photographs, some years ago by De la Rue, Loewy, and Stewart, led them to adopt the Wilsonian depression theory. Also the observations and measurements of Secchi were accepted as indicating a depression of the umbra.

From a series of drawings of sun-spots made at Palermo and Catania during a period of 11 years, M. Ricco derived results strongly in favor of the Wilsonian hypothesis, computing from especially symmetrical spots an average umbral depth of about 600 miles.

Early in 1898 Cortie, also at Stonyhurst, published the results of a comparison between Ricco's results and those obtained from

the drawings of Sidgreaves, which were made during practically the same period. Ricco gives several cases in which the penumbra was of equal width on both sides of the spot, but Cortie rightly considers such spots as favoring Howlett's theory and reduces Ricco's proportion considerably, he also shows that the Stonyhurst observations are against the depression theory. After giving the life histories of several interesting spots, and probably with the view of reconciling the contradictory observations, Cortie makes the somewhat oracular remark that, "While many spots are above the photospheric level, and many below it, it is possible that individual spots are at different levels at different periods of their life histories."

Hale remarks that the evidence derived from the observations of the apparent widths of the penumbra at various distances of the spot from the Sun's limb is too uncertain for any great reliance to be placed on it. He adds, "I am inclined to regard the advocates of the Wilsonian doctrine as having rather the better of the argument. . . . . In any case they will hardly be ready to admit that the umbra is at a higher level than the penumbra, for it can not be doubted that the penumbral filaments overlie the umbra and frequently unite to form bridges extending completely across it."

The controversy at present seems to be suspended until the evidence gathered from the observations during the present spot maximum can be published. From a consideration of the facts set forth in the above brief outline, it can be seen that no definite theory can be the outcome unless other facts are taken into consideration. The shallowness of the photosphere and the effects of refraction must have considerable influence upon the appearance of the spot as it approaches the limb.

Theories have been advanced that the sun-spots are formed of materials floating on the surface of, or above the photosphere, but the remarks of Hale quoted above seem to practically dispose of any theories of this nature.

Spectroscopic Observations. The spectroscope gives strong evidence that the sun-spots are situated below the chromosphere and reversing layer. This is indicated since the absorption due to these layers, as shown by the Fraunhofer lines, is also exerted on the spot. The earliest observations by Sir Norman Lockyer and by Professor Young showed the majority of the Fraunhofer lines absolutely unaffected in the spectrum of the sun-spot.

Neither do the faculae escape this absorption and they are manifestly above the level of the spots.

After an extended study of sun-spot spectra Cortie suggested that the spots might perhaps be at the level of the vapors of vanadium, titanium, iron and other elements having an atomic weight of about 50, since the lines of these elements are most prominent in spot spectra. But the objection has been raised that the lines of other elements whose atomic weight is considerably different from 50 are also strongly affected, as evidenced by the lines of calcium, sodium, yttrium, etc.

The reduction of Lockyer's observations showed that the lines most affected in sun-spot spectra were not in general among the chromosphere lines. In agreement with this A. Fowler of London in observations on the great spot of February 1905, shows that "high level" chromospheric lines were not among those affected in the spectrum of the spot. The writer's observations at Princeton agree with both of these. We thus have additional evidence for believing the sun-spots to be low down in the solar envelopes. It is manifest that the sun-spots are at the level of the vapors in the solar atmosphere which produce the lines most affected in the spot spectrum. The writer is inclined to the opinion that these vapors are mingled with the photospheric clouds and only extend to a short distance above them. This view is held since the most affected lines are faint, and are not present in the chromospheric spectrum; the faintness of the lines being probably caused by the thinness of the layer producing them.

The spectroheliograph offers a method of studying sun-spots which combines the advantages of the spectroscopic and the purely visual methods. It has the happy faculty of sorting out the various solar layers, and rendering it possible to take sectional photographs of the various solar phenomena. Photographs of sun-spots taken at low levels show a most striking resemblance to the ordinary appearance of the spot. While as higher levels are photographed flocculi appear, sometimes completely obscuring portions of the spot, indicating apparently a low level position for the spot.

The spectroheliograph undoubtedly promises much for the future, but as it is still in its infancy, astronomers are not absolutely sure of their interpretations of the phenomena photographed with it. However this will probably be remedied in time, and many new and interesting results are to be expected from the large instrument to be installed at Mt. Wilson.

Rotation Periods. The principle upon which this method depends, is that the portions of the Sun which are nearer the center should have the smaller velocity of rotation.

The researches of Dunér on the Sun's rotation, as determined from the displacements of certain Fraunhofer lines, by observations on advancing and receding limbs, reduced a rotation period which was longer by half a day than that deduced from the observations of spots. Since the reversing layer certainly lies above the photosphere, and according to the observations by Dunér rotates more slowly, a law of retardation with increasing distance from the Sun's center might be assumed. But this is apparently contradicted by the relations of spots and faculae; the faculae are presumably at a higher level than the spots, yet they have a shorter period of rotation, as was indicated by Stratonoff and by Wolfer.

Hence while outward *retardation* is apparently indicated by the reversing layer, outward *acceleration* is strongly suggested by faculæ, and is borne out by the more recent observations at the Yerkes Observatory on the calcium flocculi. The indications thus being, that considered in regard to their rotation periods, the order of distance from the Sun's center should be reversing layer, spots, faculae, and flocculi.

An explanation that can be offered for the discrepancy between the periods of spots and reversing layer is that Dunér's observations included only two Fraunhofer lines, and since it has been stated by Jewell that there may be a difference of several days between the rotation periods of the outer and inner portions of the Sun's atmosphere, as indicated by the displacements of different lines, it is perhaps doubtful whether Dunér's lines can be considered as giving the average rotation period of this layer. Also on account of the delicacy of the observations, and the difficulty of measuring a displacement corresponding to the small difference of rotation periods, too great reliance should not be placed on this method.

Thermal Radiation. From the thermoscopic observations by Langley and by Frost, and by Wilson in Ireland we learn that the photospheric radiation decreases as we reach the limb, while the spot radiation changes only slightly. Indeed it was found that a few spots when near the limb radiated more strongly than the neighboring photosphere.

This can be explained in one of two ways:—either the spots are high above the photosphere as maintained by Howlett and

others, and hence subject to less absorption than the photosphere; or the radiation from the spot may be a different type from that of the photosphere, therefore the absorption of the solar envelopes would probably be different for the spot and for the photosphere.

In regard to the first hypothesis, the spectroscope indicates that the absorption of the solar atmosphere is exerted on the spots as well as on the photosphere. Hence it may be reasonably asserted that the spot is at a level below the reversing layer and chromosphere.

The second hypothesis has been suggested by Prof. Young and is indicated since the solar atmosphere absorbs most strongly the short wave-length radiations of the photosphere, while these short wave-lengths are in great measure absent from the spot radiation. Thus the spot radiation would show little change due to the greater thickness of the solar atmosphere as the spot approaches the limb. It is therefore not at all inconsistent to suppose the spot to be a photospheric depression in accordance with the theory proposed by Wilson.

It is hoped that from this rather incomplete account of the various investigations that have been made, the reader will appreciate the difficulty of satisfactorily determining the level of sun-spots. The question is one of great importance and offers a fruitful field for investigators.

Princeton, N. J., June 17, 1905.

A LOST DOUBLE STAR.

ERIC DOOLITTLE.

FOR POPULAR ASTRONOMY.

The first catalogue of double stars published by Sir John Herschel contained 321 faint pairs, the angles and distances of which were estimated. Number 165 of this list was described as a 3″ pair with a position angle which would now be written 330°. The position for 1825 was, R. A. = 10^h 26.8^m, Decl. = + 12° 32′. This is practically identical with Weisse XII, 499.

The pair received no further attention until 1878, when Burnham with the 18½ inch refractor of the old Dearborn Observatory secured the measure 205°.3, 2″.59, a result which agrees well in distance with Herschel's description but which differs widely in angle.

In 1901, Burnham again put this star on his observing list

and secured the measure 330°.8, 2″.16, which was almost exactly what Herschel had described. But a year later with the 40-inch he could find no trace of the pair measured in 1901, nor, if this pair was not H 165, of the Herschel star, nor even of the star measured by him in 1878. What had become of these three pairs, if there were three, and if the measures all belonged to a single pair why could he not find it in 1902 with the 40-inch? In a most interesting article on this puzzling star, printed in Popular Astronomy for May 1903, Mr. Burnham writes:

"It is practically impossible that a 2″ pair of this kind should have any orbital movement rapid enough to make the stars apparently single in so short a time. It is certainly impossible that proper motion of both stars or either one, should have anything to do with the apparent change observed. It seems still more improbable that either component should be variable to the extent of disappearing altogether. No such double star is known in the heavens. There is no instance of a double star having vanished for any cause. It seems unlikely, to say the least. that on three occasions at widely separated intervals and with different instruments to make the setting for place, the same error should be made and the same pair, or one very similar, should be found without any trouble whatever. No entry appears in my observing book for 1878 to indicate any difficulty in finding H 165, or any correction to his place, and the same is true of the observation twenty-three years later. But after all, the trouble may be due to a series of some sort of mistakes hard to understand now, but perhaps easily explained when the star is again found. It is possible of course that this may turn out to be the long looked for variable double, and found in the recorded place where it should be according to the previous observations, and if so, it will be a most interesting pair for measurement hereafter. Should anyone find this lost, strayed or stolen member of the universe, he will please return it to the writer and no questions will be asked."

In the spring of this year upon turning the 18-inch refractor of the Flower Observatory to the exact position of the Herschel pair there at once appeared in the field a wide, quite different double which has been previously measured both by Bigourdan and Burnham. The measures of this are as follows:

1880.22	339°.0	55″.85	2*n*	Bigourdan.
1902.22	337 .5	56 .03	2*n*	Burnham.

The measures of Bigourdan were wrongly ascribed to H 165 in

the *Paris Observations* for 1883; those of Burnham were made by him while searching for H 165 and were published in Popular Astronomy, Volume 11, page 250.

This wide double is exactly where H 165 ought to be if Herschel's position is correct. But I could not find that either star of the pair was double, nor did a careful examination of all of the stars of about the eighth magnitude within 2° of the wide pair, lead to the discovery of anything at all answering to Herschel's description. I then decided to examine every star within 2° of the wide pair, even the very faint ones, and this labor was not wholly unrewarded, for $2^m\ 35^s$ preceding the wide pair and 18′ south of it there was picked up a very faint little pair which is undoubtedly the one measured by Burnham in 1878. All of the measures on this pair now stand as follows:

Pair $\alpha = \beta - =$ D M 12° (2224).
R. A. $= 10^h\ 27^m\ 11^s$; Decl. $= +\ 11°\ 56'$ (1880).

1878.28	205°.3	2″.59	8.5	8.5	1*n*, *β*.
1905.11	201 .5	2 .49	9.4	9.6	3*n*, D.
1905.41	204 .3	2 .35	9.5	9.5	1*n*, *β*.

The magnitude in the D. M. is 9.5.

Although the search was continued on several nights, nothing at all resembling H 165 was found, until it occurred to me that possibly Burnham in 1901 had inadvertently pointed his telescope 12° below the equator instead of 12° above it. I therefore set at a declination—12°, and here there was found exactly the pair which was wanted. My measures on this star were as follows:

Pair $b =$ S D $-$ 12° (3222).
R. A. $= 10^h\ 32^m\ 9^s$; Decl. $= -\ 12°\ 20'$, (1880).

1905.36	335°.0	1″.90	9.0	9.5	2*n*.

The magnitude in the S. D, is 9.1.

Here I thought, is the long lost H 165 and the mystery surrounding it is at last cleared up. But a letter from Mr. Burnham undeceived me. He wrote that he had just received Hussey's last list of new pairs, (L. O. Bulletin, No. 74), and had found that number 872 of this list was the lost H 165. The declination agrees with that of Herschel, but *the right ascension given by Herschel is just one hour in error.* All of the measures of this star are:

Pair c = H 165 = Hu. 872 = D M 12° (2070).
R. A. $= 9^h\ 31^m\ 13^s$; Decl. $= +\ 12°\ 25'$, (1880).

1825:	330° ±	3″ ±	8.0	9.0	1*n*, H.
1901.30	330.8	2 16	8.5	8.8	1*n*, *β*.
1905.16	330.7	2.41	9.0	9.5	2*n*, Hu.
1905.38	333.1	2.04	8.5	9.0	2*n*, *β*

The pair is evidently fixed.

This, I think, closes this remarkable chapter of accidents. There are three pairs which have been observed. First, the pair *a*, which was measured by Burnham in 1878 and not subsequently found on account of its extreme faintness. Then there is the pair *c*, of which Herschel gave the position one hour in error, and what is most remarkable, Burnham in setting his telescope made exactly the same error without noticing it in 1901, and by so doing happened upon exactly the star he was searching for. And finally, there is the pair *b*, whose right ascension is right and whose declination differs only in sign from that of H 165, and which within the admissable limits of error in Herschel's descriptions, agrees absolutely in magnitudes, position angle, and distance with this pair. Altogether, this is one of those strange coincidences which sometimes occur, but the theoretical probability of whose occurence is almost infinitesimal.

The Flower Observatory, June 16, 1905.

PLANET NOTES FOR SEPTEMBER AND OCTOBER, 1905.

H. C. WILSON.

Mercury will be morning star during September, making the turn of the loop in Leo (See chart in January 1905 number of Popular Astronomy,) will be at superior conjunction Oct. 12 and will be invisible during the remainder of October. The planet will be easily visible with the naked eye during the middle two weeks of September being at greatest western elongation on the morning of Sept. 15. It will then be about 18° west and 7° north of the Sun, and will also be at about its brightest for the year.

Venus is morning star during both months and will be very conspicuous toward the east, although less than half as bright as in June. The course of the planet will be southeast through Cancer and Leo. The phase will be gibbous increasing from 0.75 Sept. 1 to 0.90 Oct. 31.

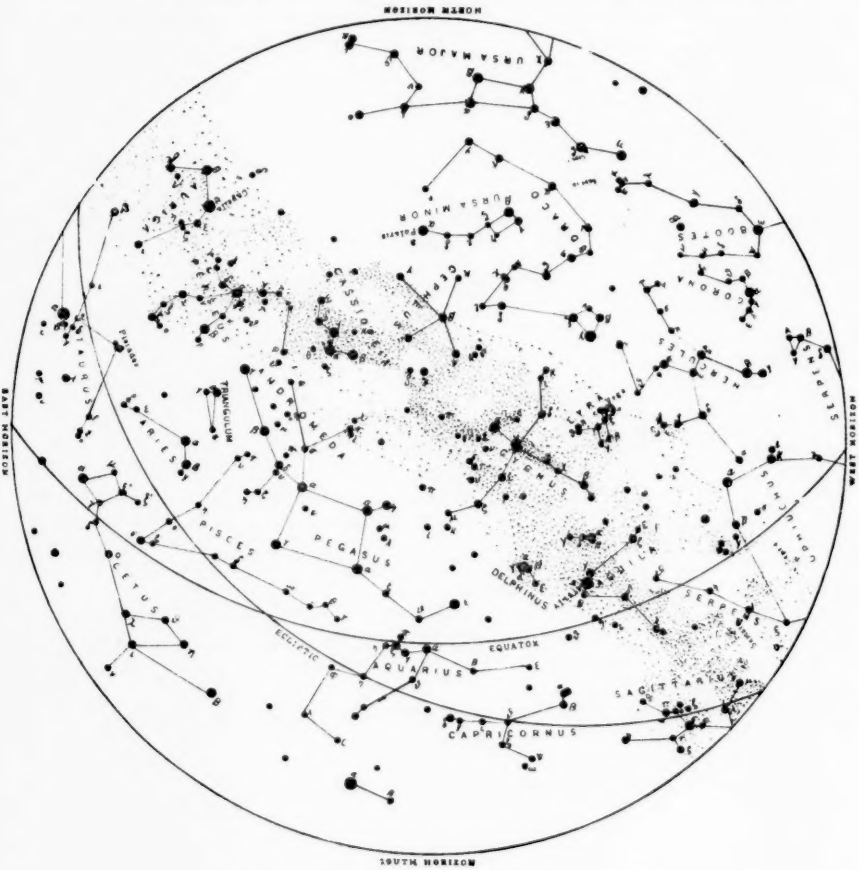
Mars may be seen in the early evening but will be at its lowest declination, so that it will be seen under the worst atmospheric conditions by observers in northern latitudes. The course of the planet is eastward through Scorpio and Sagittarius. In the evening of Oct 8, it may be of interest to note the conjunction of the two planets Mars and Uranus. The latter of the two will then be 1° 48′ north of the former, near enough so that the two may be compared as to color and brilliancy by quickly moving the telescope from the one to the other.

Jupiter has just passed quadrature and may be observed in the morning hours. He makes a conspicuous object in the morning sky, in the constellation Taurus, rivalling although not equalling Venus in brilliancy. Jupiter will be stationary in right ascension on the evening of Sept. 25, a little way southeast from the Pleiades, and after that time will move slowly toward the west.

Saturn is in best position for study for this year and will be found toward the southeast in the evening, in the constellation Aquarius. There are no bright

stars in that part of the sky, so that Saturn is easily recognized. The next brigtest star is Altair, 30° or more to the northwest from Saturn. Fomalhaut is about 20° to the southeast from the planet.

Uranus will be at quadrature, 90° east from the Sun, Sept. 23 and may be found with the aid of a telescope in the early evening. It is among the faint



THE CONSTELLATIONS AT 9 P. M. OCTOBER 1, 1905.

stars in the Milky Way in Sagittarius, about 5° west and 2° north of the star λ. Its motion will be a little over one degree westward during the two months.

Neptune will be at quadrature, 90° west from the Sun, Oct. 3 and so may be observed in the morning hours. It may be found with the aid of a telescope, about midway between δ and μ Geminorum. For its position among the faint stars see chart in the February 1905 number of Popular Astronomy.

Occultations Visible at Washington.

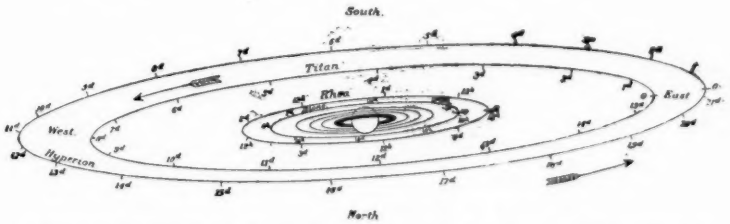
Date. 1905.	Star's Name.	Magnitude.	IMMERSION. Washington M.T. h m	IMMERSION. Angle from N pt. °	EMERSION. Washington M. T. h m	EMERSION. Angle from N pt. °	Duration. h m
Sept. 2	B. A. C. 4647	6.1	8 38	140	9 24	252	0 46
4	B. A. C. 5188	6.5	5 16	52	6 02	343	0 46
8	Mayer 814	6.1	7 41	33	8 37	314	0 56
12	h^1 Aquarii	5.4	10 38	28	11 50	279	1 12
13	29 Piscium	5.1	7 43	125	18 17	194	0 34
14	14 Ceti	5.4	8 36	98	9 39	214	1 03
19	δ^2 Tauri	3.6	9 11	65	10 04	266	0 53
19	δ^1 Tauri	4.2	9 16	41	10 00	290	0 44
19	81 Tauri	5.5	10 20	155	10 29	174	0 09
19	Bradley 619	4.8	10 31	355	11 42	334	1 11
19	85 Tauri	6.0	10 39	116	11 25	211	0 46
19	B. A. C. 1406	6.5	11 47	31	12 37	295	0 50
19	α Tauri	1.1	13 10	25	14 03	298	0 53
20	115 Tauri	5.3	13 09	53	14 16	279	1 07
Oct. 5	B. A. C. 6616	6.4	7 29	31	8 23	310	0 54
7	29 Capricorni	5.5	11 36	82	12 42	238	1 06
12	f Piscium	5.3	16 14	99	17 12	221	0 58
13	Lalande 3830	6.5	17 25	38	18 18	289	0 53
15	D. M. + 12° 473	6.2	7 23	90	8 19	232	0 56

Satellites of Saturn.

[Central Standard Time; the hours are reckoned from noon E. = East Elongation; I. = Inferior Conjunction (south of planet;) W. = West Elongation; S. = Superior Conjunction (north of planet.)]

I. Mimas. Period 0^d $22.^h6$.

Sept.	1	15.1 W.	Sept.	11	12.5 E.	Sept.	22	8.6 W.	Oct.	6	11.9 W.
	2	13.7 W.		12	11.2 E.		23	7.2 W.		7	10.5 W.
	3	12.3 W.		13	9.8 E.		27	13.0 E.		8	9.1 W.
	4	10.9 W.		14	8.4 E.		28	11.6 E.		9	7.7 W.
	5	9.5 W.		15	7.0 E.		29	10.2 E.		10	6.3 W.
	6	8.1 W.		19	12.8 W.		30	8.8 E.		11	4.9 W.
	7	6.8 W.		20	11.4 W.	Oct.	1	7.5 E.			
	10	13.9 E.		21	10.0 W.		2	6.1 E.			



Apparent orbits of the Seven Inner Satellites of Saturn at Opposition in 1905 as seen in an Inverting Telescope.

II. Enceladus. Period 1^d $8.^h9$.

Sept.	2	2.2 E.	Sept.	13	1.2 E.	Sept.	24	0.2 E.	Oct.	4	23.3 E.
	3	11.0 E.		14	10.1 E.		25	9.1 E.		6	8.1 E.
	4	19.9 E.		15	18.9 E.		26	18.0 E.		7	17.0 E.
	6	4.8 E.		17	3.8 E.		28	2.9 E.		9	1.9 E.
	7	13.7 E.		18	12.7 E.		29	11.7 E.		10	10.8 E.
	8	22.5 E.		19	21.6 E.		30	20.6 E.		11	19.7 E.
	10	7.4 E.		21	6.5 E.	Oct.	2	5.5 E.		13	4.6 E.
	11	16.3 E.		22	15.3 E.		3	14.6 E.		14	13.4 E.

III. Tethys. Period $1^d\ 21.^h3$.

Sept.	2	1.7 E.	Sept.	13	9.5 E.	Sept	24	17.3 E.	Oct.	6	1.1 E.
	3	23.0 E.		15	6.8 E.		26	14.6 E.		7	22.4 E.
	5	20.3 E.		17	4.1 E.		28	11.9 E.		9	19.7 E.
	7	17.6 E.		19	1.4 E.		30	9.2 E.		11	17.0 E.
	9	14.9 E.		20	22.7 E.	Oct.	2	6.5 E.		13	14.3 E.
	11	12.2 E.		22	20.0 E.		4	3.8 E.		15	11.6 E.

IV. Dione. Period $2^d\ 17.^h7$.

Sept.	1	15.8 E.	Sept.	12	14.4 E.	Sept.	23	13.1 E.	Oct.	4	11.7 E.
	4	9.4 E.		15	8.0 E.		26	6.7 E.		7	5.3 E.
	7	3.1 E.		18	1.7 E.		29	0.4 E.		9	23.0 E.
	9	20.7 E.		20	19.4 E.	Oct	1	18.0 E.		12	16.7 E.
										15	10.4 E.

V. Rhea. Period $4^d\ 12.^h5$.

Sept.	3	20.0 E.	Sept.	17	9.0 E.	Sept.	30	22.1 E.	Oct.	9	22.8 E.
	8	8.3 E.		21	21.4 E.	Oct.	5	10.5 E.		14	11.2 E.
	12	20.7 E.		26	9.7 E.						

VI. Titan. Period $15^d\ 23.^h3$.

Sept.	2	10.1 W.	Sept.	14	9.6 I.	Sept.	26	4.3 E.	Oct.	8	1.5 S.
	6	5.9 S.		18	7.8 W.		30	7.5 I.		11	2.3 E.
	10	6.5 E.		22	3.6 S.	Oct.	4	5.7 W.			

VII. Hyperion. Period $21^d\ 7.^h6$.

Sept.	3	.8 W.	Sept.	14	.4 I.	Sept.	25	.2 S.	Oct.	5	.8 I.
	8	.8 E.		20	.2 W.		30	.1 E.		10	.5 W.

VIII. Iapetus. Period $79^d\ 22.^h1$.

Aug.	22	.4 E.	Sept.	11	.0 I.	Oct.	1	.2 W.	Oct.	20	.7 S.

COMET AND ASTEROID NOTES.

New Asteroids.—The following have been added to the list of new minor planets since our last note:

	Discovered by	at	Local Mean Time	R. A.	Decl.	Mag.
			h m	h m	° ′	
1905 QO	Götz	Heidelberg	May 28 12 20.5	16 21.8	−19 45	11.8
QP	Götz	"	28 12 20.5	16 30.0	−24 14	12.5

Names of New Asteroids.—Three of the asteroids discovered in 1902 have lately been named as follows:—

Number	Provisional Designation	Name	Discover
(485)	1902 HZ	Genua	Carneva
(486)	1902 JB	Cremona	Carneva
(494)	1902 JV	Wolf	Virtus

Elliptic Elements of Comet 1905 a.—In *A. N.* 4023 Mr. A Wedemeyer gives elliptic elements of this comet determined from Observations made March 26, April 8 and April 28

$T = 1905$ April 4.0991 Berlin mean time.

$\omega = 358^\circ\ 13'\ 20''.3$

$\Omega = 157\ 23\ 27\ .8$ 1905.0

$i = 40\ 14\ 38\ .4$

$\log e = 9.988506$

$\log q = 0.047307$

$\log a = 1.630354$

‡Period of 279 Years.

In *A. N.* 4027 Mr. T. Banachiewicz gives similar elements with a period of 200.62 years.

VARIABLE STARS.

Minima of Variable Stars of the Algol Type.

[Given to the nearest hour in Greenwich Mean Time. To reduce to Central Standard time subtract 6 hours, or for Eastern time subtract 5 hours.]

U Cephei.

	d	h
Sept.	2	13
	5	1
	7	12
	10	0
	12	12
	15	0
	17	12
	20	0
	22	11
	24	23
	27	11
	29	23
Oct.	2	11
	4	23
	7	10
	9	22
	12	10
	14	22

Z Persei.

	d	h
Sept.	2	18
	5	19
	8	20
	11	22
	14	23
	18	0
	21	2
	24	3
	27	5
	30	6
Oct.	3	7
	6	9
	9	10
	12	11
	15	13

Algol.

	d	h
Sept.	2	7
	5	13
	8	10
	11	7
	14	4
	17	1
	19	21
	22	18
	25	15
	28	12
Oct.	1	9
	4	6
	7	2
	9	23
	12	20
	15	17

RT Persei.

	d	h
Sept.	1	9
	2	5
	3	1
	3	22
	4	18
	5	14
	6	11
	7	7
	8	4
	9	0
	9	20
	10	17
	11	13
	12	10
	13	6
	14	2
	14	23
	15	19
	16	16
	17	12
	18	8
	19	5
	20	1
	20	21
	21	18
	22	14
	23	11
	24	7
	25	3
	25	24
	26	20
	27	17

RT Persei.

	d	h
Sept.	28	13
	29	9
	30	6
Oct.	1	2
	1	22
	2	19
	3	15
	4	12
	5	8
	6	4
	7	1
	7	21
	8	18
	9	14
	10	10
	11	7
	12	3
	12	23
	13	20
	14	16
	15	13

λ Tauri.

	d	h
Sept.	2	8
	6	7
	10	6
	14	4
	18	3
	22	2
	26	1
	30	0
Oct.	3	23
	7	22
	11	21
	15	19

R Canis Maj.

	d	h
Sept.	9	8
	10	12
	11	15
	12	18
	13	22
	15	1
	16	4
	17	7
	18	11
	19	14
	20	17
	21	20
	23	0
	24	3
	25	6
	26	9
	27	13
	28	16
	29	19
	30	22
Oct.	2	2
	3	5
	4	8
	5	12
	6	15
	7	18
	8	21
	10	1
	11	4
	12	7
	13	10
	14	14
	15	17

Y Camelop.

	d	h
Sept.	4	3
	7	11
	10	18
	14	1
	17	9
	20	16
	23	23

Y Camelop.

	d	h
Sept.	27	6
	30	14
Oct.	3	21
	7	5
	10	12
	13	19

RR Puppis.

	d	h
Sept.	3	8
	9	18
	16	4
	22	15
	29	1
Oct.	5	11
	11	22

V Puppis.

	d	h
Sept.	1	22
	3	9
	4	20
	6	7
	7	18
	9	5
	10	16
	12	3
	13	14
	15	1

V Puppis.

	d	h
Sept.	16	11
	17	22
	19	9
	20	20
	22	7
	23	18
	25	5
	26	16
	28	3
	29	14
Oct.	1	1
	2	11
	3	22
	5	9
	6	20
	8	7
	9	18
	11	5
	12	16
	14	3
	15	14

S Cancri.

	d	h
	6	23
	16	11
	25	22
Oct.	5	10
	14	22

W Urs. Maj.

	d	h
Sept.	1-30	18
Oct.	1-31	19

Z Draconis.

	d	h
Sept.	1	23
	3	8
	4	16
	6	1
	7	9
	8	18
	10	2
	11	11
	12	20
	14	4
	15	13
	16	21
	18	6
	19	14
	20	23
	22	8
	23	16
	25	1
	26	9
	27	18
	29	3
	30	11
Oct.	1	20
	3	4
	4	13
	5	21
	7	6
	8	15
	9	23
	11	8
	12	16
	14	1
	15	9

δ Libræ.

	d	h
Sept.	3	4
	5	12
	7	19
	10	3
	12	11
	14	19
	17	3
	19	11
	21	19
	24	2
	26	10
	28	18
Oct.	1	2
	3	10
	5	18
	8	2
	10	9
	12	17
	15	1

Minima of Variable Stars of the Algol Type.—Continued.

U Coronæ.

	d	h
Sept.	4	4
	7	15
	11	2
	14	13
	17	23
	21	10
	24	21
	28	8
Oct.	1	19
	5	6
	8	17
	12	3
	15	14

R Aræ.

	d	h
Sept.	2	20
	7	6
	11	16
	16	3
	20	13
	24	23
	29	9
Oct.	3	19
	8	6
	12	16

U Ophiuchi.

	d	h
Sept.	1	18
	2	14
	3	10
	4	6
	5	3
	5	23
	6	19
	7	15
	8	11
	9	7
	10	3
	10	23
	11	20
	12	16
	13	12
	14	8
	15	4
	16	0
	16	20
	17	17
	18	13
	19	9
	20	5
	21	1
	21	21
	22	17
	23	13
	24	10
	25	6
	26	2

U Ophiuchi.

	d	h
Sept.	26	22
	27	18
	28	14
	29	10
	30	6
Oct.	1	3
	1	23
	2	19
	3	15
	4	11
	5	7
	6	3
	6	23
	7	20
	8	16
	9	12
	10	8
	11	4
	12	0
	12	20
	13	16
	14	13
	15	8

Z Herculis

	d	h
Sept.	2	2
	3	23
	6	2
	7	23
	10	2
	11	23
	14	2
	15	23
	18	2
	19	23
	22	1
	23	22
	26	1
	27	22
	30	1
Oct.	1	22
	4	1
	5	22
	8	1
	9	22
	12	1
	13	22

RS Sagittarii.

	d	h
Sept.	1	22
	4	8
	6	18
	9	4
	11	14
	14	0
	16	10
	18	20
	21	6
	23	16
	26	2

RS Sagittarii.

	d	h
Sept.	28	12
	30	22
Oct.	3	8
	5	18
	8	4
	10	14
	13	0
	15	10

V Serpentis.

	d	h
Sept.	3	16
	7	3
	10	14
	14	1
	17	12
	20	23
	24	9
	27	20
Oct.	1	7
	4	18
	8	5
	11	16
	15	3

RX Herculis.

	d	h
Sept.	1	8
	2	5
	3	3
	4	0
	4	21
	5	19
	6	16
	7	13
	8	11
	9	9
	10	5
	11	3
	12	0
	12	22
	13	19
	14	16
	15	14
	16	11
	17	8
	18	6
	19	3
	20	0
	20	22
	22	16
	23	14
	24	11
	25	8
	26	6
	27	3
	28	0
	28	22
	29	19
	30	16
Oct.	1	14
	2	11

RX Herculis.

	d	h
Oct.	3	8
	4	6
	5	3
	6	0
	6	22
	7	19
	8	16
	9	14
	10	11
	11	8
	12	6
	13	3
	14	1
	14	22
	15	19

RV Lyræ.

	d	h
Sept.	1	13
	5	3
	8	18
	12	8
	15	23
	19	13
	23	3
	26	18
	30	8
Oct.	3	22
	7	13
	11	3
	14	18

U Sagittæ.

	d	h
Sept.	4	6
	7	15
	11	0
	14	9
	17	19
	21	4
	24	13
	27	22
Oct.	1	7
	4	16
	8	1
	11	11
	14	19

SY Cygni.

	d	h
Sept.	6	6
	12	6
	18	6
	24	6
	30	6
Oct.	6	7
	12	7
	18	7

WW Cygni.

	d	h
Sept.	1	20
	5	4
	8	11
	11	19
	15	3
	18	10
	21	18
	25	1
	28	9
Oct.	1	17
	5	0
	8	8
	11	16
	14	23

SW Cygni.

	d	h
Sept.	2	19
	7	9
	11	22
	16	12
	22	2
	25	16
	30	5
Oct.	4	19
	9	9
	13	23

VW Cygni.

	d	h
Sept.	1	16
	10	2
	18	13
	26	23
Oct.	5	10
	13	20

UW Cygni.

	d	h
Sept.	2	16
	6	3
	9	13
	13	0
	16	11
	19	22
	23	9
	26	19
	30	6
Oct.	3	17
	7	4
	10	15
	14	2

W Delphini.

	d	h
Sept.	3	6
	8	1
	12	22
	17	16
	22	11
	27	7
Oct.	2	2
	6	22
	11	17

Minima of Variable Stars of the Algol Type.—Continued.

Y Cygni.		Y Cygni.		Y Cygni.		VV Cygni.		VV Cygni.	
d	h	d	h	d	h	d	h	d	h
Sept. 2	3	Sept. 21	12	Oct. 11	2	Sept. 11	17	Sept. 30	22
3	13	23	3	12	11	13	5	Oct. 2	9
5	3	24	12	14	2	14	16	3	21
6	12	26	2	15	11	16	3	5	8
8	3	27	12	VV Cygni.		17	15	6	20
9	12	29	2	Sept. 1	9	19	2	8	7
11	3	30	12	2	20	20	14	9	19
12	12	Oct. 2	2	4	7	22	1	11	6
14	3	3	12	5	19	23	13	12	18
15	12	5	2	7	7	25	0	14	5
17	3	6	12	8	18	26	12	UZ Cygni.	
18	12	8	2	10	6	27	23	Oct. 21	
20	3	9	12			29	11		

Approximate Magnitudes of Variable Stars on Aug. 10, 1905.

[Communicated by the Director of Harvard College Observatory, Cambridge, Mass.]

Name.	R. A. 1900. h m	Decl. 1900. ° ′	Magn.		Name.	R. A. 1900. h m	Decl. 1900. ° ′	Magn.	
T Androm.	0 17.2	+26 26		*f*	R Camelop.	14 25.1	+84 17	10.3	*d*
T Cassiop.	0 17.8	+55 14	10.8	*d*	R Bootis	14 32.8	+27 10	6.5	
R Androm.	0 18.8	+38 1	10.6	*d*	S Librae	15 15.6	−20 2	8.5	
S Ceti	0 19.0	− 9 53		*s*	S Serpentis	15 17.0	+14 40	10.5	*i*
S Cassiop.	1 12.3	+72 5	10.5	*d*	S Coronae	15 17.3	+31 44	11.8	*d*
R Piscium	1 25.5	+ 2 22	8		S Urs. Min.	15 33.4	+78 58	9.3	*i*
U Persei	1 52.9	+54 20	7.5	*i*	R Coronae	15 44.4	+28 28	7.2	*i*
R Arietis	2 10.4	+24 36	8.5		V "	15 45.9	+39 52	1.0	*d*
o Ceti	2 14.3	− 3 26	9		R Serpentis	15 46.1	+15 26	8	
S Persei	2 15.7	+58 8	9		R Herculis	16 1.7	+18 38	10	*i*
R Ceti	2 20.9	− 0 38		*u*	R Scorpii	16 11.7	−22 42	12.7	*d*
U "	2 28.9	−13 35		*s*	S "	16 11.7	−22 39	12.5	*i*
R Trianguli	2 31.0	+33 50	8	*d*	U Herculis	16 21.4	+19 7	10.5	*d*
R Persei	3 23.7	+35 20		*u*	W Herculis	16 31.7	+38 32	10	*i*
R Tauri	4 22.8	+ 9 56		*s*	R Draconis	16 32.4	+66 58	8.5	*d*
S "	4 23.7	+ 9 44		*s*	S Herculis	16 47.4	+15 7	8.5	*d*
R Aurigæ	5 9.2	+53 28	10	*d*	R Ophiuchi	17 2.0	−15 58	7	
U Orionis	5 49.9	+20 10		*s*	T Herculis	18 5.3	+31 0	10.5	i
R Lyncis	6 53.0	+55 28		*f*	R Scuti	18 42.2	− 5 49	6.5	*d*
R Gemin.	7 1.3	+22 52		*s*	R Aquilae	19 1.6	+ 8 5	10.2	*d*
S Canis Min.	7 27.3	+ 8 32		*s*	R Sagittarii	19 10.8	−19 59	7	
R Cancri	8 11.0	+12 2		*s*	S "	19 13.6	−19 12	13	
V "	8 16.0	+17 36		*s*	R Cygni	19 14.1	+49 58	13	*d*
S Hydrae	8 48.4	+ 3 27		*s*	RT "	19 40.8	+48 32	12	*d*
T "	8 50.8	− 8 46		*s*	X "	19 46.7	+32 40	12	*d*
R Leo. Min.	9 39.6	+34 58		*f*	S "	20 3.4	+57 42		*f*
R Leonis	9 42.2	+11 54	7.5	*d*	RS "	20 9.8	+38 28	9.6	*d*
R Urs. Maj.	10 37.6	+69 18	10.5	*d*	R Delphini	20 10.1	+ 8 47	12	*d*
R Comae Ber.	11 59.1	+19 20	9	*i*	U Cygni	20 16.5	+47 35	8	*i*
T Virginis	12 9.5	− 5 29		*f*	V "	20 38.1	+47 47	11	
R Corvi	12 14.4	−18 42		*s*	T Aquarii	20 44.7	− 5 31	11.8	*d*
Y Virginis	12 28.7	− 3 52	11.8	*d*	R Vulpec.	20 59.9	+23 26	9.5	*d*
T Urs. Maj.	12 31.8	+60 2	13	*d*	T Cephei	21 8.2	+68 5	7.5	*d*
R Virginis	12 33.4	+ 7 32	9.0	*i*	S "	21 36.5	+78 10	9	
S Urs. Maj.	12 39.6	+61 38	7.8	*i*	S Lacertae	22 24.6	+39 48	12	
U Virginis	12 46.0	+ 6 6	9.1	*d*	R "	22 38.8	+41 51	13	*d*
V "	13 22.6	− 2 39		*f*	S Aquarii	22 51.8	−20 53	14	
R Hydrae	13 24.2	−22 46	6.8	*d*	R Pegasi	23 1.6	+10 0	8	
S Virginis	13 27.8	− 6 41	9.5	*i*	S "	23 15.5	+ 8 22	9	
R Can. Ven.	13 44.6	+40 2	9.8	*d*	R Aquarii	23 38.6	+15 50		*u*
S Bootis	14 19.5	+54 16	10.3	*d*	R Cassiop.	23 53.3	+50 50	9.3	*d*

Approximate Magnitudes of Variable Stars.—Continued.

NOTE:—*f* denotes that the variable is probably fainter than the magnitude 13; *i*, that the light is increasing; *d*, that the light is decreasing; *s*, that it is near the Sun; and *u*, that its magnitude is unknown.

From observations made at the Eadie, McCormick, Vassar College and Harvard Observatories.

Variable Stars of Short Period not of the Algol Type.

	Minimum.		Maximum.			Minimum.		Maximum.	
	d	h	d	h		d	h	d	h
T Velorum	Sept. 1	11	Sept. 2	20	Y Sagittarii	Sept. 12	1	Sept. 14	3
U Sagittarii	1	12	4	12	SU Cygni	12	5	13	13
T Vulpeculae	2	5	3	14	V Velorum	12	5	13	4
VY Cygni	2	15	4	17	W Sagittarii	12	12	15	12
R Triang. Austr.	2	16	3	16	S Triang. Austr.	12	15	14	17
RV Scorpii	2	20	4	5	R Triang. Austr.	12	19	13	19
S Muscae	3	7	6	18	S Muscae	13	0	16	11
V Velorum	3	12	4	11	β Lyrae	13	0	16	7
X Sagittarii	3	13	6	10	S Normae	13	8	17	17
S Normae	3	14	7	23	U Vulpeculae	13	22	16	1
η Aquilae	3	20	6	5	S Crucis	14	1	15	13
T Crucis	4	1	6	2	S Sagittae	14	9	17	19
δ Cephei	4	3	5	12	ζ Geminorum	14	14	19	14
V Carinae	4	10	6	14	W Geminorum	14	20	17	11
δ Geminorum	4	10	9	11	δ Cephei	14	23	16	18
SU Cygni	4	13	5	21	RV Scorpii	14	23	16	8
U Aquilæ	4	15	6	19	U Sagittarii	15	0	18	0
S Crucis	4	16	6	4	W Virginis	15	4	23	8
R Crucis	4	20	6	5	T Velorum	15	9	16	18
W Sagittarii	4	22	7	22	T Vulpeculæ	15	13	16	22
Y Ophiuchi	5	7	11	12	SU Cygni	16	1	17	9
U Vulpeculae	5	23	8	2	R Triang. Austr.	16	5	17	5
S Sagittae	6	0	9	10	R Crucis	16	11	17	20
R Triang. Austr.	6	1	7	1	V Velorum	16	14	17	13
T Velorum	6	2	7	11	V Centauri	17	6	18	17
V Centauri	6	7	7	18	VY Cygni	17	8	19	10
S Triang. Austr.	6	7	8	9	T Crucis	17	12	19	13
Y Sagittarii	6	7	8	9	X Sagittarii	17	14	20	11
β Lyræ	6	13	9	15	κ Pavonis	17	18	22	3
T Vulpeculae	6	16	8	1	V Carinae	17	19	19	23
W Geminorum	7	2	9	17	Y Sagittarii	17	21	19	23
V Velorum	7	21	8	20	η Aquilae	18	4	20	13
U Sagittarii	8	6	11	5	U Aquilae	18	17	20	21
SU Cygni	8	9	9	17	S Crucis	18	18	20	6
TX Cygni	8	10	13	13	S Triang. Austr.	18	23	21	1
κ Pavonis	8	15	13	0	T Monocerotis	19	2	27	0
RV Scorpii	8	21	10	6	β Lyræ	19	11	22	13
S Crucis	9	9	10	21	R Triang. Austr.	19	14	20	14
R Triang. Austr.	9	10	11	10	SU Cygni	19	22	21	6
δ Cephei	9	12	10	21	T Vulpeculae	20	0	21	9
X Cygni	10	0	15	3	T Velorum	20	1	21	10
VY Cygni	10	12	12	14	W Sagittarii	20	2	23	2
X Sagittarii	10	13	13	10	δ Cephei	20	6	21	15
R Crucis	10	16	12	11	V Velorum	20	23	21	22
T Crucis	10	18	12	19	RV Scorpii	21	0	22	9
T Velorum	10	18	12	3	U Sagittarii	21	18	24	18
η Aquilae	11	0	13	9	U Vulpeculae	21	21	24	0
V Carinæ	11	2	13	6	R Crucis	22	7	23	16
T Vulpeculae	11	2	12	11	Y Ophiuchi	22	10	28	15
U Aquilae	11	15	13	19	W Geminorum	22	13	25	3
V Centauri	11	18	13	5	S Muscae	22	15	26	2

Variable Stars of Short Period not of the Algol Type.—(Continued.)

	Minimum.		Maximum.			Minimum.		Maximum.	
	d	h	d	h		d	h	d	h
V Centauri Sept.	22	18	24	5	VY Cygni Oct.	4	1	Oct. 6	3
S Sagittae	22	18	27	4	V Velorum	4	2	5	1
R Triang. Austr.	23	0	24	0	ζ Geminorum	4	21	9	21
S Normae	23	2	27	11	Y Sagittarii	5	3	7	5
TX Cygni	23	4	28	7	U Sagittarii	5	5	8	5
S Crucis	23	11	24	23	W Sagittarii	5	7	8	7
Y Sagittarii	23	15	25	17	SU Cygni	5	7	6	15
SU Cygni	23	18	25	2	κ Pavonis	5	22	10	7
T Crucis	24	5	26	6	δ Cephei	6	8	7	17
T Vulpeculæ	24	10	25	19	R Triang. Austr.	6	13	7	13
V Carinæ	24	11	26	15	S Crucis	7	13	9	1
X Sagittarii	24	14	27	11	T Crucis	7	16	9	17
T Velorum	24	16	26	1	T Vulpeculae	7	17	9	2
ζ Geminorum	24	18	29	18	U Vulpeculae	7	20	9	23
S Triang. Austr.	25	6	27	8	V Carinae	7	21	10	1
V Velorum	25	8	26	7	TX Cygni	7	21	13	0
η Aquilae	25	8	27	17	S Triang. Austr.	7	22	10	0
δ Cephei	25	15	27	0	W Geminorum	8	1	10	16
U Aquilae	25	17	27	21	V Velorum	8	11	9	10
β Lyræ	25	22	29	5	T Velorum	8	14	9	23
VY Cygni	26	5	28	7	X Sagittarii	8	14	11	11
R Triang. Austr.	26	9	27	9	β Lyrae	8	20	12	3
X Cygni	26	9	31	12	SU Cygni	9	3	10	11
κ Pavonis	26	20	31	5	RV Scorpii	9	5	10	14
RV Scorpii	27	2	28	11	V Centauri	9	6	10	17
SU Cygni	27	14	28	22	S Sagittae	9	12	12	22
W Sagittarii	27	17	30	17	Y Ophiuchi	9	13	15	18
R Crucis	28	3	29	12	η Aquilae	9	17	12	1
S Crucis	28	4	29	16	R Crucis	9	18	11	3
V Centauri	28	6	29	17	U Aqluae	9	18	11	22
U Sagittarii	28	11	31	10	R Triang Austr.	9	22	10	22
T Vulpeculae	28	20	30	5	Y Sagittarii	22	10	13	0
T Velorum	29	7	30	16	δ Cephei	11	17	13	2
Y Sagittarii	29	9	31	23	VY Cygni	11	22	14	0
V Velorum	29	17	30	16	S Muscae	11	22	15	9
R Triang. Austr.	29	18	30	18	U Sagittarii	11	23	14	22
U Vulpeculae	29	21	Oct. 2	0	T Vulpeculae	12	4	13	13
W Geminorum	30	7	2	22	S Crucis	12	5	13	17
δ Cephei	30	23	2	8	S Normae	12	14	16	23
T Crucis	30	23	3	0	X Cygni	12	19	17	22
S Sagittae Oct.	1	3	4	13	V Velorum	12	20	13	19
V Carinae	1	5	3	9	W Sagittarii	12	22	15	22
SU Cygni	1	10	2	18	SU Cygni	13	0	14	8
S Triang. Austr.	1	14	3	16	T Velorum	13	5	14	14
X Sagittarii	1	14	4	11	R Triang. Austr.	13	8	14	8
S Muscae	2	7	5	18	S Triang. Austr.	14	6	16	8
β Lyræ	2	9	5	11	T Crucis	14	10	16	11
η Aquilae	2	13	4	22	V Carinae	14	14	16	18
U Aquilae	2	18	4	22	V Centauri	14	18	16	5
S Crucis	2	20	4	8	κ Pavonis	15	0	19	9
S Normae	2	20	7	5	ζ Geminorum	15	1	20	1
R Triang. Austr.	3	3	4	3	RV Scorpii	15	6	16	16
RV Scorpii	3	3	4	12	β Lyrae	15	7	18	9
T Vulpeculæ	3	7	4	16	R Crucis	15	14	16	23
V Centauri	3	18	5	5	X Sagittarii	15	15	18	12
T Velorum	3	22	5	7	W Geminorum	15	19	18	10
R Crucis	3	23	5	8	U Vulpeculae	15	20	17	23

Maxima of Y Lyræ.

Period $12^h\ 03^m.9$. The minimum occurs $1^h\ 40^m$ before the maximum.

	d	h	d	h	d	h	d	h
Sept.	1- 9	21	Sept. 18-25	23	Oct. 1- 3	0	Oct. 12-18	2
	10-17	22	26-30	0	4-11	1		

Maxima of U Y Cygni.

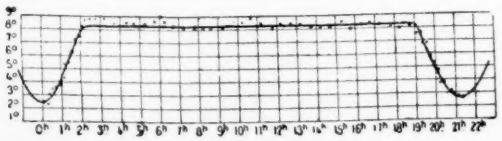
Period $13^h\ 27^m\ 27^s.59$. The minimum occurs $1^h\ 55^m$ before the maximum.

	d	h	d	h	d	h	d	h
Sept.	1	6	Sept. 12	11	Sept. 23	16	Oct. 4	22
	2	9	13	14	24	19	6	1
	3	12	14	17	25	22	7	3
	4	15	15	20	27	1	8	6
	5	18	16	23	28	4	9	9
	6	21	18	2	29	7	10	12
	8	0	19	5	30	10	11	15
	9	3	20	8	Oct. 1	13	12	18
	10	5	21	11	2	16	13	21
	11	8	22	14	3	19	15	0

Maxima of RZ Lyræ.

Period $12^h\ 16^m\ 15^s.0$.

	d	h	d	h	d	h	d	h
Sept.	1	18	Sept. 13	0	Sept. 24	6	Oct. 5	12
	2	19	14	1	25	7	6	13
	3	19	15	1	26	7	7	13
	4	20	16	2	27	8	8	14
	5	21	17	3	28	8	9	14
	6	21	18	3	29	9	10	15
	7	22	19	4	30	10	11	16
	8	22	20	4	Oct. 1	10	12	16
	9	23	21	5	2	11	13	17
	10	23	22	5	3	11	14	17
	12	0	23	6	4	12	15	18



LIGHT CURVE OF RX HERCULIS.

Variable Star RX Herculis.—In the *Bulletin Astronomique* for June 1905 Mr. M. Luizet gives a new determination of the elements of this variable based upon observations extending from May 1899 to Nov. 1904. He finds the star to be of the Algol type with a period of $21^h\ 20^m\ 34^s.5$. The formula for the prediction of minima is

Minimum = Julian Day 2414566.5369 (Paris m. t.) + 0.889288 E
= Oct. 3, 1898, $12^h\ 3^m$ (Paris m. t.) + $21^h\ 20^m\ 34^s.5$ E.

The variation of light is regular, waning for $2^h\ 20^m$, then increasing for $2^h\ 15^m$, and remaining constant for about $16^h\ 45^m$. The amplitude of variation is about 0.6 magnitude (maximum 7.0 and minimum 7.6.)

Elements of the Variable S Sagittae.—In *A. N.* 4030 Mr. M. Luizet gives new elements of the variable star S Sagittae based upon observations by Gore from 1885 to 1890 and Luizet 1898 to 1904. The star has two maxima separated by a secondary minimum, the maxima being about two days apart. The depression in the light curve due to the secondary minimum is very small but appears to be quite certain. The new period obtained by Luizet is $8^d\ 9^h\ 10^m\ 13^s = 8.38209$ days.

Minimum = J. D. 2409860.37 (Paris m. t. + 8.38209 (E − 389)
or 1885 Nov. 14 $8^h\ 53^m$ + $8^d\ 9^h\ 10^m\ 13^s$ (E − 389).

Variable Star 47. 1905 Aurigae.—In *A. N.* 40121 Mr. A. Stanley Williams gives the following approximate elements of this star:

Maximum = 1905 April 6.3 (J. D. 2416942.3) + $3^d.75$ E.

„The variation is of the δ Cephei type, the increase being considerably quicker than the decrease, the interval M − m being 1.1 days. The range of variation is about 0.55 magnitude (5.0 to 5.55.)

New Variables 48—57. 1905 Aquilæ.—These are all in the vicinity of the star λ Aquilæ, and were discovered by M. and G. Wolf on photographs taken at Heidelberg. Their positions are as follows:

	R.A. 1900			Decl. 1900			Range of Variation Noted.		
	h	m	s	°	′	″	m		m
48. 1905	19	32	16.94	+ 8	58	38.2	12.5	—	14.5
49		34	31.43	+ 11	57	46.7	12.5	—	14.5
50		39	01.17	+ 10	38	20.6	12.5	—	13.5
51		39	40.53	+ 7	20	43.3	13	—	<14.5
52		40	18.08	+ 11	31	12.2	13	—	15.
53		42	43.22	+ 8	19	58.8	12	—	14.5
54		42	46.83	+ 9	08	11.5	13.5	—	<15
55		43	12.46	+ 12	53	44.5	13	—	14
56		49	01.98	+ 12	32	12.8	12	—	14.5
57. 1905	19	49	23.86	+ 10	02	20.6	12.5	—	13.5

Charts of all these are given in *A. N.* 4018.

New Variables 61. 1905 Persei and 62. 1905 Aurigæ.—These are announced by Prof. Ceraski in *A. N.* 4016 and were discovered on the Moscow photographs by Mme. L. Ceraski. Their positions are

	R. A. 1855			Decl. 1855		R. A. 1900			Decl. 1900	
	h	m	s	°	′	h	m	s	°	′
61	4	1	15.2	+ 33	52	4	04	08.7	+ 33	59.4
62	5	30	20	+ 37	33	5	33	24	+ 37	35

The first (61. 1905 Persei) on plates taken in 1899, 1904 and 1905 ranges in magnitude from 9.5 to 11.0 and the period must be not greater than two months. The second (62. 1905 Aurigæ) is 10.0, at brightest noted on plates taken between 1895 and 1905 and below 12.0 at minimum. The period is unknown.

New Variable 63. 1905 Geminorum.—Mr. A. Tass in *A. N.* 4028 finds that the star BD, 22° 1576 is variable. This star has been used as a comparison star for the long period variable R Geminorum. In the BD its magnitude is given as 8.4; in the Potsdam photometric *Durchmusterung* it is 6.33' while Mr. Tass on a number of dates in 1904-5 found the photometric magnitude to vary between 8.0 and 9.4. The position for 1855 is

R. A. $6^h\ 58^m\ 34^s.0$; Decl. + 22° 44'.0.

New Variable 72. 1905 Cygni.—This was discovered by M. and G. Wolf on a photograph taken May 29, 1905 when its brightness was between 9.5^m and 10^m. On older plates, which show stars of the fifteenth magnitude, no trace of the variable appears. Its position for 1855 is

R. A. $19^h\ 43.7^m + 41°\ 17'$.

Three New Variables 74, 75 and 76. 1905.—These are announced by Professor W. Ceraski in *A. N.* 4028 as discovered on the Moscow photographs by Mme. L. Ceraski. Their positions are

	R. A. 1855			Decl. 1855		R. A. 1900			Decl. 1900	
	h	m	s	°	′	h	m	s	°	′
74. 1905 Draconis	18	22	14	+ 72	38.2	18	21	21	+ 72	39.6
75. 1905 Draconis	18	17	29	+ 59	30.8	18	18	05	+ 59	32.0
76. 1905 Cygni	19	29	30	+ 56	04.6	19	30	25	+ 56	10.4

The first varies between about 10.4^m and $< 11.7^m$, the second between 9.4^m, and 12.5^m, the third between about 8.6^m and 9.3^m, so far as observed.

New Variable 77. 1905 Ophiuchi.—This was discovered by Rev. Thomas D. Anderson, of Northrig, Haddington, Scotland, with a three-inch refractor. On May 8, 11, 22 and 30 of this year the star was not visible, while a tenth magnitude star near by was always seen, but on June 30 it was brighter than its neighbor and was estimated at 9.9^m. The star is BD. + 3°, 3656 and is given as of 8.6^m in the BD. Its position for 1855 is

R. A. $18^h\ 9^m\ 25^s.8$; Decl. + 3° 38′ 49″.

GENERAL NOTES.

In consequence of the considerable rise in the prices of paper and labor, the management of this publication is obliged to raise the subscription price of this publication, from $2.50 to $3.50. Part of this increased cost is due to the addition of eight pages of matter, regularly above that we have usually given. Much good matter has been furnished recently that we could not use on account of lack of room for it.

The call for some stronger articles by students of astronomy as well as the kind offer of this needed help by some of the ablest astronomers in the United States encourages us to try again to meet a real need in some of the lines of the older Astronomy that is pressing in upon us from various ways. We believe the additional eight pages monthly will materially aid in both directions.

In this same line, it is a noticeable fact, that students of late years are not giving as much attention to the study of mathematical astronomy as they have given in years past. This is a serious mistake; somehow it must be remedied. Mathematics in astronomy is fundamental, and the thorough and complete and continued study of the various branches of mathematical analysis that are needed in astronomy must be suitably provided for. Chicago University and the University of California are doing splendid work of this kind. More of it is greatly needed to keep the older astronomy abreast with the progress of science in other departments.

The Janssen Medal of the French Astronomical Society was awarded to Professor Percival Lowell in 1904 for his work on Mars. This is noteworthy

from the fact that it is the first and only medal ever awarded for the study and interpretation of the surface features of Mars. Mr. Lowell has carried forward this work on Mars for years with remarkable zeal, fidelity and success and he is worthy of the conspicuous honor he has received in the award of the Janssen Medal.

The Sixth Satellite of Jupiter.—A telegram from Lick Observatory July 30 announces that Albrecht has observed the sixth satellite of Jupiter with the Crossley Reflector on three nights in July. The following positions of the satellite with reference to the planet were determined.

	Position Angle	Distance
	°	′
July 25.95	55.0	25.1
26.97	52.7	24.3
27.93	50.7	23.6

Elements of Jupiter's Seventh Satellite.—In Lick Observatory Bulletin, No. 82, Mr. Frank E. Ross gives the results of a preliminary determination of the elements of Jupiter's seventh Satellite, from observations secured by Perrine with the Crossley Reflector at Lick Observatory on Jan. 3. Feb. 8 and March 6, 1905.

ECLIPTIC ELEMENTS.

Mean Jovicentric Longitude at Epoch	333°.55	1905 Jan. 0.0 Greenwich Mean Time
Longitude of Perijove	336 .65	
Longitude of Node	237 .23	
Inclination to Ecliptic	31 .0	
Inclination to Jupiter's Orbit	32 .0	
Longitude of Node on Jupiter's Orbit	238 .6	

ELEMENTS REFERRED TO EARTH'S EQUATOR.

Mean Jovicentric Right Ascension	328°.18	1905 Jan. 0.0 Greenwich Mean Time
Right Ascension of Perijove	331 .28	
Right Ascension of Node	281 .13	
Inclination to Equator	26 .2	

Mean Daily Motion 1°.358

Log $a = 8.9004$ $\quad a = 52'.54$ (for $\log \Delta = 0.71624$)

$e = 0.0246$ $\quad$ Period = 265.0 days

Distance at maximum elongation 70′.

EPHEMERIS OF JUPITER'S SEVENTH SATELLITE.

Greenwich Mean Noon	Position Angle	Distance	Greenwich Mean Noon	Position Angle	Distance
	°	′		°	′
Sept. 4	292	49	Oct. 14	289	55
9	292	53	19	288	51
14	292	56	24	287	46
19	291	58	29	287	40
24	291	59	Nov. 3	285	33
29	290	59	8	284	26
Oct. 4	290	59	13	283	18
9	289	58			

Seventh Satellite of Jupiter.—According to a telegram received August 15 from Lick Observatory, Dr. Albrecht observed the new satellite with the Crossley reflector August 7.96. Its position angle was then 289.°7 and its distance 54.′6. It was observed also on Aug. 9 and 10.

C. D. Perrine, Astronomer at Lick Observatory Married. Who can say that lucky stars do not govern the fate of men sometimes? On July 4, 1905, our esteemed friend Charles Dillon Perrine and Miss Belle Smith were married at the home of the bride's sister in Philadelphia. They went to New York, and as members of the private eclipse expedition party, sent out from Lick Observatory, sailed for Spain.

To most of our readers Mr. Perrine is known as an astronomer at Lick Observatory who has earned for himself an enviable reputation during the last twelve years in his chosen profession at that great Observatory. During this time he has made a study of comets, observed eclipses, and made some notable discoveries. His list of newly discovered comets now numbers 13, but his recent discovery of two new satellites of Jupiter is the most surprising and notable of all his work in this direction.

He has been a member of three Solar Eclipse expeditions; in 1889, in California in 1900 in Georgia; in 1901 he was in charge of the Lick Expedition to observe the total solar eclipse in the island of Sumatra, and now he is in the fourth expedition to observe the total solar eclipse in Spain Aug. 29-30, 1905. Mr. Perrine has also been awarded the Lalande Prize from the French Academy of Science for his services to astronomy.

Shadow Bands in the Total Eclipse of the Sun, Aug. 30, 1905. A. Lawrence Rotch, of Blue Hill Observatory, Hyde Park, Mass., has issued a Circular calling the attention of observers of the coming total eclipse of the Sun to five points, in regard to which he seeks information: (1) The direction in which the bands lie, (2) The direction in which they move (3) The velocity with which they move (4) The width of the bands, (5) Their distance apart. All of these aspects of the shadow bands are likely to be different before and after the total phase of the eclipse so that two sets of observations are needed for best results.

Mr. Rotch calls attention to the care that should be exercised in making these observations in regard to time, width and speed of the bands while they are visible. Attention is also called to the fact that some have claimed to see them during the time of the total phase of the eclipse.

Professor D. Todd Observes Total Eclipse in Tripoli. From a letter recently received it is learned that Professor David Todd of Amherst College was to sail for Naples on July 27 1905, thence to Tripoli in Africa which point he expects to reach about Aug. 10. He takes with him a 12-inch photographic lens for Corona work, two three inch photographic lenses, duplicates of the Leck lenses for observing the intramercurial regions, one four inch Georz lens for rapid photography of the Corona which is expected to give 200 exposures during the 189 seconds of time that the total phase of the eclipse will last at Tripoli. He will also try to photograph the shadow bands, and to make visual observations of the Corona with and without a telescope.

Carl Axel Robert Lundin: Scientific expert in cutting and fashioning glasses of great telescopes. He has done important work on the large objectives of Russia, of the Lick and Yerkes observatories, and lately on the 18-inch objective of the Amherst College observatory, which is wholly his work. In 1854 Amherst conferred the degree of master of arts on Alvan Clark, who had built our first telescope. The same degree, for a similar service, is conferred on his successor, who has kept pace with the progress of astronomical science.

Geo. A. Hill of the United States Naval Observatory, author of the article elsewhere given in this issue is a member of the Eclipse party from Washington which sailed for Europe about the first of July. The preparations for this expedition are said to be very thorough and very complete. Great results are hoped for from the new 7.5-inch, 65 feet focus photoheliograph which has been in the course of preparation for some time past.

Halley's Comet: Its Past History and 1910 Return. The above is the title of a short bibliography with notes by Eugene Fairfield McPike on this important comet. The pamphlet in hand consists of five pages and is a reprint from the Smithsonian Miscellaneous Collections, (Quarterly Issue) Vol. 48 part 1. These notes will be useful in the near future to those who wish to make a study of Halley's Comet.

Some Refinements of Mechanical Science is the title of an address of Ambrose Swasey, of Warner and Swasey, Cleveland, Ohio, delivered March 13, 1905, before the American Society of Mechanical Engineers. This interesting address considers the division and measurement of time, the construction of clocks and the performance of some of the best modern astronomical time pieces, the division of circles for the measurement of angles, the screw for the micrometer, the spider threads, and the ruling of diffraction gratings ranging from 14,000 to 20,000 lines, to the inch, with an error in ruling which does not exceed one millionth of an inch.

To give some idea of the refinement of measurement which has been reached in mechanical science, Mr. Swasey refers to the work of Mr. J. A. Brashear and Professor Michaelson on the production of plane surfaces and exact standards of length. The problem recently given to Mr. Brashear was to make a plane surface on speculum metal that should be as nearly perfect as he could make it. He succeeded in producing such a surface that was in error only one four hundred thousandth of an inch.

Professor Michaelson conceived the idea that standards of length should be measured by wave lengths of light to be permanent in the case of destruction. By the aid of Mr. Brashear, this work has been so successful "that Professor Michaelson was invited to continue it at the Bureau of Weights and Measures at Sèvres, France, where the standard meter was kept in an underground vault and inspected only at long intervals or when needed for important tests. The final result of the experiments which occupied nearly a year shows that there are 1,553,164.5 wave lengths of red cadmium light in the French standard meter at 15° centigrade. So great is the accuracy of these experiments that they can be repeated within one part in two millions." This shows something of the marvelous exactness of mechanical science in dividing and measuring linear values of any kind.

Lawrence Fellowship, Dept. of Astronomy at Indiana University. It is interesting to learn that Percival Lowell has established and endowed a Fellowship in Astronomy at Indiana University. The Fellows are appointed by the department of the University subject to the approval of Mr. Lowell, and their practical study of Astronomy is carried forward at Lowell Observatory. John C. Duncan has received the appointment for the years 1905-6.

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